Gruppaning ko‘pxillikka ta’siri

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Kirish:Ushbu tezisda Gruppaning ko‘pxillakka ta’siri o‘rganilgan bo‘lib silliqligi, erkinligi, xosliklari o‘rganilgan.Faktor fazo orbitalari o‘rganilgan va teoremaga ko‘ra silindr va Myobius yaprog‘i tekislikka lokal diffeomorf.

Tarif. Silliq M ko‘pxillikka G gruppaning ta‘siri deb

$$a : G \times M \to M, \quad a(g, x) = g \cdot x$$

akslanishtirishga aytiladi.

Bunday ta‘sir xossalari quyidagilar:

1. Agar $\forall\ x \in M$ va $e \in G$, G gruppasining birlik elementi uchun $e \cdot x = x$ bajarilsa ta‘sir erkin deyiladi.

2. Har bir $g \in G$ uchun
$$a_g : M \to M, \quad x \mapsto g \cdot x$$
silliq bo‘lsa, ta‘sir silliq deyiladi.

3. M ko‘pxilligida G gruppaning ta‘siri xos deyiladi, agar har qanday $x, y \in M$ nuqtalar uchun shunday U_x, U_y ochiq atroflari mavjud bo‘ladiki,unda:
$$U_x \bigcap gU_y \neq \emptyset$$

4. Faktor-fazo M/G to‘plami orbitlardan iborat oila kabi aniqlanadi:
$$M/G = \{[x] \mid x \in M\}, \quad [x] = G \cdot x.$$
Unda kanonik proyeksiya aniqlangan bo‘ladi:
$$\pi : M \to M/G, \quad \pi(x) = [x].$$

5. M/G faktor-topologiyada $U \subset M/G$ ochiq bo‘lishi uchun
$$\pi^{-1}(U) \subset M$$
teskari ham ochiq bo‘lishi zarur va yetarli.

Teorema: Silliq m o‘lchamli M ko‘pxillikka G gruppaning ta’sirida silliq, erkin va xos bo‘lsa, $M \setminus G$ da yagona differentsiallanuvchi struktura mavjud bo‘lib, unga nisbatan $\pi : M \to M \setminus G$ kanonik proyeksiya lokal diffeomarfizm bo‘ladi.

Misol: Quyidagi diffeomorfizmlarni ko‘rib chiqaylik. $a, b : R^2 \to R^2$ akslanishlar quyidacha berilgan:
$a(x, y) = (x, y + 1) \qquad b(x, y) = (x + 0.5, -y)$

G_a, G_b- mos ravishda $\{a\}$ va $\{b\}$ diffeomorfizmlar hosil qilgan gruppalar boʻlsa, ular R^2 da gruppa taʻsirini aniqlaydi.
Bu ta’sirlar natijasida silindr va Myobius yaprogʻi hosil boʻladi. Yuqoridagi teoremaga koʻra ular tekislikka lokal diffeomorf.

Adabiyotlar

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A note path in the n-fold symmetric product of the space X

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All of our space are Hausdorff unless otherwise indicated. The symbol N stands for the set of positive integers and R stands for the set of real numbers. Given a space X, we define its hyperspaces as the following sets:
1) $CL(X) = \{A \subset X \mid A \text{ is closed and nonempty }\}$;
2) $2^X = \{A \in CL(X) \mid A \text{ is compact }\}$;
3) $\mathcal{F}_n(X) = \{A \in 2^X \mid A \text{ has at most } n \text{ points }\}$, $n \in N$ (see [1, 2]).
$CL(X)$ is topologized by the Vietoris topology defined as the topology generated by

$$\beta = \{\langle U_1, ..., U_k\rangle \mid U_1, ..., U_k \text{ are open subsets of } X,\ k \in N\},$$

where $\langle U_1, ..., U_k\rangle = \{A \in CL(X) \mid A \subset \bigcup U_j \text{ and } A \cap U_j \neq \emptyset \text{ for each } j \in \{1, ..., k\}\}$.
Note that, by definition, 2^X, $\mathcal{F}_n(X)$ and $\mathcal{F}(X)$ are subsets of $CL(X)$. Hence, they are topologized with the appropriate restriction of the Vietoris topology. Moreover,
1) $CL(X)$ is called the *hyperspace of nonempty closed subsets of* X;
2) 2^X is called the *hyperspace of nonempty compact subsets of* X;
3) $\mathcal{F}_n(X)$ is called the *$n-$fold symmetric product of* X;
4) $\mathcal{F}(X)$ is called the *hyperspace of finite subsets of* X.
On the other hand, it is obvious that $\mathcal{F}(X) = \bigcup_{n=1}^{\infty} \mathcal{F}_n(X)$ and $\mathcal{F}_n(X) \subset \mathcal{F}_{n+1}(X)$ for each $n \in N$ (see [1, 2]).

A continuous mapping $f : [0, 1] \to X$ is called a *path* in X. The point $f(0)$ is called the initial point and $f(1)$ is called the final or terminal point of this path. If $x \in X$, then one defines $e_x : I \to X$ as the constant path, i.e. $e_x(t) = x$ for any $t \in I$. A topological space X is said to be *path-connected* if given any two points x_0, x_1 in X there is a path in X from x_0 to x_1.

Theorem. If the mapping $f : I \to X$ is path in X from the point x_0 to the point x_1, then a mapping $\mathcal{F}_n f : I \to \mathcal{F}_n X$ defined by $\mathcal{F}_n f(t) = \bigcap\{A \in \mathcal{F}_n X : f(t) \in A,\ \ \forall t \in [0, 1]\}$ is a path from the point $A_0 = \bigcap\{A \in \mathcal{F}_n : f(0) \in A\}$ to the point $A_1 = \bigcap\{A \in \mathcal{F}_n : f(1) \in A\}$ in $\mathcal{F}_n X$.

Corollary. If a topological space X is path-connected, then the space $\mathcal{F}_n X$ is also path-connected.