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## ABOUT SOLUTIONS OF SOME PARTIAL INTEGRAL EQUATIONS OF THE FREDHOLM TYPE

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**Annotation:** We study the solvability of the Fredholm partial integral equations of second type with degenerate kernels in space  $L_2(\Omega_1 \times \Omega_2)$ .

**Keywords:** Kaplansky-Hilbert module, Fredholm determinants, Nonhomogeneous and Homogeneous partial integral equation of Fredholm type.

## FREDHOLM TURIDAGI BA'ZI QISM INTEGRAL TENGLAMALARNING YECHIMLARI HAQIDA

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**Annotatsiya:** Biz  $L_2(\Omega_1 \times \Omega_2)$  fazoda yadrolari ajralgan ikkinchi tur Fredholm qismaniy integral tenglamalarining yechiluvchanligini o'rganamiz.

**Kalit so'zlar:** Kaplanskiy-Hilbert moduli, Fredholm determinantlari, Fredholm tipidagi bir jinsli va bir jinsli bo'lmagan qismaniy integral tenglamalar.

## О РАЗРЕШИМОСТЬ ОДНОГО ЧАСТИЧНО ИНТЕГРАЛЬНОГО УРАВНЕНИЙ ТИПА ФРЕДГОЛЬМА

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**Аннотация.** Исследуется разрешимость частично интегрального уравнения типа Фредгольма второго рода с вырожденным ядром в пространстве  $L_2(\Omega_1 \times \Omega_2)$ .

**Ключевые слова:** Модуль Капланского-Гильберта, определители Фредгольма, неоднородное и однородное частично интегральных уравнений типа Фредгольма.

### Introduction

Let  $T_1 \subset \mathbb{R}^{V_1}$  and  $S \subset \mathbb{R}^{V_2}$  be sets of finite Lebesgue measure. On some ideal function space  $X$  measurable functions  $f(x, y)$  given by  $T \times S$ , consider the following equation:

$$A_0 f(x, y) = (A_1 + A_2 + A_{1,2}) f(x, y) + g(x, y) \quad (1)$$

where  $g$  is a given function, operators  $A_\alpha, \alpha \in \{0, 1, 2, \{1, 2\}\}$ , represent space  $X$  into itself and at the same time:  $A_0$  is the operator of multiplication by a function  $h_0(x, y)$ , i.e.  $A_0 f(x, y) = h_0(x, y) f(x, y)$ ,  $A_1$  is an integral operator in which when acting on an unknown function  $f(x, y)$ , the integration is taken

only the first argument,  $A_2$  is an integral operator, in which when acting on an unknown function  $f(x, y)$ , the integration is taken only the second argument,  $A_{1,2}$  is an integral operator in which when acting on an unknown function  $f(x, y)$ , the integration is taken and on the first and on the second arguments.

Equation (1) is called an equation with partial integrals or just a partial integral equation. If in equation (1) in each integral operator  $A_1, A_2$  and  $A$  integration is expressed as a definite integral, then the equation (1) is called a partially integral equation of Fredholm type, and the corresponding operator  $A_0 - (A_1 + A_2 + A_{1,2})$  is called partially Fredholm integral operator.

### Analysis of literature

In 1932 V.I. Romanovsky [1] in his article when studying problems of the theory of Markov chains investigated the solvability of the partial integral equation of Fredholm type in the space of continuous functions. However, it must be emphasized that in many literary sources, the work of Abdus Salam [2] is cited as the first source on theory of partially integral operators of Fredholm type (for example, see [3], [4]).

V.I. Romanovsky investigated the integral equation

$$U(x, y) = f(x, y) + \lambda \int_a^b U(t, x) \varphi(t, x, y) dt$$

in the space of continuous functions  $C[a, b]^2$  by a method similar to the method Fredholm determinants [1,5], here  $f(x, y) \in C[a, b]^2$  is a well-known function,  $\varphi(t, x, y)$  is the kernel of the integral equation,  $U(x, y)$  is an unknown function, and  $\lambda$  - parameter of the equation. This integral equation sometimes is called a partially integral equation of Romanovsky type of the second kind, and the integral operator in this equation is called the partially Romanovsky integral operator (see [6-9]).

Note that linear operators with partial integrals of Fredholm type, acting in space  $L_2$ , can be realized as measurable bundle of compact operators

Linear operators and partial integral equations in different classes of function spaces were studied in monographs [10-12]

Many equations arising in the mechanics of continuous media, at solving problems of evolutionary type, in the theory of elasticity and theory viscoelasticity, solid mechanics and in other areas of physics and mechanics, represented in the form of partially integral equations Volterra-Fredholm type of the second kind [11,13]

### Research methodology

In [11] the applications of partial integral equations and operators in solving problems of continuous mechanics, elasticity problems and other problems were considered. Until now, a solvability of PIE in the space  $L_2$  is left open. The present paper is devoted this problem. Namely, we investigate a solvability of problem for PIE with degenerate kernels from the class  $L_2$ .

### Analyzes and results

Let  $(\Omega, \mathcal{E}, \mu)$  be a measurable space with a finite measure,  $L_0(\Omega)$  be the  $*$  - algebra of equivalence classes of all complex measurable functions on  $\Omega$  and  $L_\infty(\Omega) \subset L_0(\Omega)$  - subalgebra of equivalence classes of all bounded measurable functions on  $\Omega$ . By denote [f] equivalence classes of function on  $f \in L_0(\Omega)$ .

We denote by  $L_\infty[L_2(\Omega_1), \Omega_2]$  the set of all complex measurable functions  $f(x, y)$  on  $\Omega_1 \times \Omega_2$  satisfying the condition: the integral

$$\varphi_0(y) = \int_{\Omega_1} |f(x, y)|^2 d\mu_1(x)$$

exists for almost all  $y \in \Omega_2$  and  $\varphi_0 \in L_\infty(\Omega_2)$ . In the  $L_\infty[L_2(\Omega_1), \Omega_2]$  we define  $L_\infty(\Omega_2)$  - valued inner product  $\langle f, g \rangle$ :

$$\langle f, g \rangle = \langle f, g \rangle (y) = \int_{\Omega_1} f(x, y) \overline{g(x, y)} d\mu_1(x).$$

Let  $\varphi_1, \varphi_2 \in L_\infty[L_2(\Omega_1)]$ ,  $\psi \in C(\Omega_2)$  and

$$k_1(x, s, y) = \varphi_1(x)\overline{\varphi_1(s)}\psi(y) + \varphi_2(x)\overline{\varphi_2(s)}\psi(y), (x, s, y) \in \Omega_1^2 \times \Omega_2.$$

Then the partial integral operator (PIO)  $T_1$  :

$$T_1 f(x, y) = \int_{\Omega_1} k_1(x, s, y) f(s, y) d\mu_1(s)$$

is bounded linear operator on the  $L_2(\Omega_1 \times \Omega_2)$ .

In this paper we study the solvability of the partial integral equation

$$f - T_1 f = g, \tag{2}$$

in space  $L_2(\Omega_1 \times \Omega_2)$ , where  $g = g(x, y) \in L_2(\Omega_1 \times \Omega_2)$  is a given function.

**1. About solution of nonhomogeneous Fredholm partial integral equation with degenerate kernel.**

If  $g(x, y) \neq \theta$ , the partial integral equation (2) is called the *nonhomogeneous Fredholm partial integral equation* (NPIE) of second type with degenerate kernel. The homogeneous partial integral equation (HPIE) corresponding the NPIE (2) has the following form

$$h - T_1 h = \theta. \tag{3}$$

We define measurable functions  $\tau_{11}, \tau_{12}, \tau_{21}, \tau_{22}$  on  $\Omega_2$  by

$$\begin{aligned} \tau_{11}(\omega) &= \int_{\Omega_1} \varphi_1(s) \overline{\varphi_1(s)} \psi(\omega) d\mu_1(s), \quad \tau_{21}(\omega) = \int_{\Omega_1} \alpha_2 \varphi_2(s) \overline{\varphi_1(s)} \psi(\omega) d\mu_1(s), \\ \tau_{12}(\omega) &= \int_{\Omega_1} \alpha_1 \varphi_1(s) \overline{\varphi_2(s)} \psi(\omega) d\mu_1(s), \quad \tau_{22}(\omega) = \int_{\Omega_1} \alpha_2 \varphi_2(s) \overline{\varphi_2(s)} \psi(\omega) d\mu_1(s) \end{aligned}$$

One can see that  $\tau_{11}, \tau_{12}, \tau_{21}, \tau_{22} \in L_\infty(\Omega_2)$ .

Let  $e$  be an identity element of the algebra  $L_\infty(\Omega_2)$ , i.e.  $e(\omega) = 1$  for almost all  $\omega \in \Omega_2$ .

Now we define  $2 \times 2$  matrices  $T$  and  $I$  with the elements of the  $L_0(\Omega_2)$  as follows

$$T = T(\omega) = \begin{pmatrix} \tau_{11}(\omega) & \tau_{21}(\omega) \\ \tau_{12}(\omega) & \tau_{22}(\omega) \end{pmatrix}, \quad I = I(\omega) = \begin{pmatrix} e(\omega) & \theta(\omega) \\ \theta(\omega) & e(\omega) \end{pmatrix}$$

Let  $D_1(\omega)$  be a function on  $\Omega_2$  given by

$$D_1(\omega) = \det(T(\omega) - I(\omega)), \quad \omega \in \Omega_2.$$

One can see that  $D_1(\omega)$  is a measurable function. Moreover  $D_1 = D_1(\omega) \in L_\infty(\Omega_2)$ . The function  $D_1$  is called a *determinant Fredholm* of the PIE (2).

Let  $\phi \in L_0(\Omega)$ . We define its support by the equality  $s(\phi) = s_\phi = [\chi_{\Omega(\phi \neq 0)}]$ , where  $L_0[L_2(\Omega_1), \Omega_2]$ .

**Lemma1.** If  $s(D_1) = e$  (i.e.  $D_1(\omega) \neq 0$  for almost all  $\omega \in \Omega_2$ ), then the HPIE (3) has in the  $L_2(\Omega_1 \times \Omega_2)$  trivial solution.

**Proof.** Suppose that  $h(x, y)$  is a solution HPIE (3). Let us denote

$$\begin{cases} b_1(\omega) = \int_{\Omega_1} \overline{\varphi_1(s)} \psi(\omega) h(s, \omega) d\mu_1(s), \\ b_2(\omega) = \int_{\Omega_1} \overline{\varphi_2(s)} \psi(\omega) h(s, \omega) d\mu_1(s) \end{cases} \tag{4}$$

Obviously,  $b_1, b_2 \in L_2(\Omega_2)$  and

$$h(x, y) = \alpha_1 b_1(y) \varphi_1(x) + \alpha_2 b_2(y) \varphi_2(x). \tag{5}$$

From the equality (4) and (5) we obtain the following system of equations for the unknown functions  $b_1(y), b_2(y)$ :

$$\begin{cases} b_1(y) = \tau_{11}(y)b_1(y) + \tau_{21}(y)b_2(y), \\ b_2(y) = \tau_{12}(y)b_1(y) + \tau_{22}(y)b_2(y) \end{cases}$$

Consequently,

$$\begin{cases} (\tau_{11}(y) - 1)b_1(y) + \tau_{12}(y)b_2(y) = \theta(y), \\ \tau_{21}(y)b_1(y) + (\tau_{22}(y) - 1)b_2(y) = \theta(y). \end{cases} \tag{6}$$

We can write the system equations (6) in matrix form as following:

$$(\mathbf{T} - \mathbf{I})\mathbf{b} = \theta, \tag{7}$$

where  $\mathbf{b} = \mathbf{b}(\omega)$  columns of the matrix which consist of functions  $b_{11} = b_1(y), b_{21} = b_2(y)$ .

Let  $s_{D_1} = e$ . Then  $D_1(\omega) \neq 0$  for almost all  $\omega \in \Omega_2$ . It follows that, for almost all  $y \in \Omega_2$  equations (6) has only a trivial solution, i.e., the equation (7) has only zero solution:  $\mathbf{b} = \theta$ .

**Theorem1.** If  $s(D_1) = e$  (i.e.  $D_1(\omega) \neq 0$  for almost all  $\omega \in \Omega_2$ ), then the NPIE (2) in  $L_0[L_2(\Omega_1)]$  has unique solution. An arbitrary solution of the NPIE (2) has the form

$$f(x, y) = g(x, y) + b_1(y)\varphi_1(x) + b_2(y)\varphi_2(x),$$

where  $b_1(y) = \int_{\Omega_1} \overline{\varphi_1(s)} \psi(y) f(s, y) d\mu_1(s), b_2(y) = \int_{\Omega_1} \overline{\varphi_2(s, y)} \psi(y) f(s, y) d\mu_1(s)$

**Proof.** Let

$$\begin{cases} a_1(\omega) = \int_{\Omega_1} \overline{\varphi_1(s)} \psi(\omega) g(s, \omega) d\mu_1(s), \\ a_2(\omega) = \int_{\Omega_1} \overline{\varphi_2(s, \omega)} \psi(\omega) g(s, \omega) d\mu_1(s) \end{cases}$$

It is clear that  $a_1, a_2 \in L_2(\Omega_2)$ . For unknown functions  $b_1(y), b_2(y)$  we obtain the system of equations

$$\begin{cases} a_1(\omega) = (1 - \tau_{11}(\omega))b_1(\omega) - \tau_{21}(\omega)b_2(\omega), \\ a_2(\omega) = -\tau_{12}(\omega)b_1(\omega) + (1 - \tau_{22}(\omega))b_2(\omega) \end{cases}$$

Put  $D_1(\omega) = D_1(\omega) = \begin{vmatrix} 1 - \tau_{11}(\omega) & -\tau_{21}(\omega) \\ -\tau_{12}(\omega) & 1 - \tau_{22}(\omega) \end{vmatrix}$ .

i) if  $s(a_1) = \theta, s(a_2) = \theta$ , then  $g(x, y) = \theta$  is a solution of the HPIE (3) and there is no another solution of the equation (3).

ii) Suppose that  $s(a_1) \neq \theta$  or  $s(a_2) \neq \theta$ . We define measurable functions  $\Delta_1(\omega), \Delta_2(\omega)$  on  $\Omega_2$  as follows: the elements in 1st and 2nd columns of the determinant  $\det(\mathbf{I}(\omega) - \mathbf{T}(\omega))$  replace by the functions  $a_1(\omega), a_2(\omega)$  and the resulting determinant we denote by  $1 - \tau_{22}(\omega) a_1(\omega)$

$$\Delta_1(\omega) = \begin{vmatrix} a_1(\omega) & -\tau_{12}(\omega) \\ a_2(\omega) & 1 - \tau_{22}(\omega) \end{vmatrix}, \Delta_2(\omega) = \begin{vmatrix} 1 - \tau_{22}(\omega) & a_1(\omega) \\ -\tau_{21}(\omega) & a_2(\omega) \end{vmatrix}$$

It is easy to see that  $\Delta_1, \Delta_2 \in L_2(\Omega_2)$ .

Let  $\omega \in \Omega_2$  is a fixed element. In space  $L_2(\Omega_1)$  we consider the Fredholm second type equation

$$\varphi(x) - (K_\omega \varphi)(x) = g(x, \omega), \tag{8}$$

where  $K_{\omega}\varphi(x) = \int_{\Omega_1} k_1(x, s, \omega)\varphi(s)d\mu_1(s)$ .

The equation (8) for every  $\omega \in \Omega_2$  has a unique solution

$$\varphi(x) = \varphi_{\omega}(x) = g(x, \omega) + \frac{\Delta_1(\omega)}{D_1(\omega)}\varphi_1(x, \omega) + \frac{\Delta_2(\omega)}{D_1(\omega)}\varphi_2(x, \omega).$$

Clearly, the function

$$f_0(x, y) = g(x, y) + \frac{\Delta_1(y)}{D_1(y)}\varphi_1(x, y) + \frac{\Delta_2(y)}{D_1(y)}\varphi_2(x, y) \tag{9}$$

belongs to  $L_0[L_2(\Omega_1), \Omega_2]$  and

$$f_0 - T_1f_0 = g,$$

i.e.  $f_0$  is a solution of (2).

Now we prove the uniqueness of the solution  $f_0$  of the equation (2). Let  $f_1 \in L_0[L_2(\Omega_2)]$  is a solution of the equation (2) and  $f_1 \neq f_0$ . Then from the equality  $f_1 - T_1f_1 = g$  for a.e.  $\omega \in \Omega_2$  we get

$$f_1(x, \omega) - (T_1f_1)(x, \omega) = g(x, \omega),$$

i.e.

$$f_1(x, \omega) - (K_{\omega}f_1)(x, \omega) = g(x, \omega) \text{ a.e. } \omega \in \Omega_2.$$

By the uniqueness of the solution of the Fredholm equation (8), we obtain

$$f_1(x, \omega) = g(x, \omega) + \frac{\Delta_1(\omega)}{D_1(\omega)}\varphi_1(x, \omega) + \frac{\Delta_2(\omega)}{D_1(\omega)}\varphi_2(x, \omega)$$

for a.e.  $\omega \in \Omega_2$ , i.e.  $f_1(x, y) = f_0(x, y)$ .

**2. About solution of homogeneous Fredholm partial integral equation with degenerate kernel.**

We define linear operators  $A_1$  and  $A_2$ :

$$A_1f(x, y) = \begin{vmatrix} a_{f,1}(y) & -\tau_{12}(y) \\ a_{f,2}(y) & 1 - \tau_{22}(y) \end{vmatrix}, A_2f(x, y) = \begin{vmatrix} 1 - \tau_{22}(y) & a_{f,1}(y) \\ -\tau_{21}(y) & a_{f,2}(y) \end{vmatrix}$$

where

$$a_{f,1}(\omega) = \int_{\Omega_1} \overline{\varphi_1(s)}\psi(\omega)f(s, \omega)d\mu_1(s), a_{f,2}(\omega) = \int_{\Omega_1} \overline{\varphi_2(s)}\psi_2(\omega)f(s, \omega)d\mu_1(s).$$

Note that  $A_1$  and  $A_2$  are linear bounded operators in  $L_2(\Omega_1 \times \Omega_2)$  and they are PIO. By Theorem1 and equality (9), we get the following theorem.

**Theorem2.** Let  $s(D_1) = e$ . If  $D_1^{-1} \in L_{\infty}(\Omega_2)$ , then the operator  $I - T_1$  ( $I$  is the identity operator) is invertible and

$$(I - T_1)^{-1}f(x, y) = f(x, y) + \frac{1}{D_1(y)}(S_1f)(x, y),$$

where

$$S_1f(x, y) = \varphi_1(x)A_1f(x, y) + \varphi_2(x)A_2f(x, y), f \in L_2(\Omega_1 \times \Omega_2).$$

**Proof.** Let  $s(D_1) \neq e$ . Then HPIE (3) has a nontrivial solution in the  $L_2(\Omega_1 \times \Omega_2)$  and moreover, any solution  $h(x, y)$  of the equation (3) has the form

$$h(x, y) = (e(y) - s_{D_1}(y))\alpha_1b_1(y)\varphi_1(x) + (e(y) - s_{D_1}(y))\alpha_2b_2(y)\varphi_2(x),$$

where  $b_1, b_2 \in L_2(\Omega_2)$  are arbitraries.

Let  $h \in L_2(\Omega_1 \times \Omega_2)$  be a solution of the equation (3). Then the function  $h$  has the form

$$h(x, y) = \alpha_1 d_1(y) \varphi_1(x) + \alpha_2 d_2(y) \varphi_2(x),$$

where  $d_1, d_2 \in L_2(\Omega_2)$  are arbitraries. Let  $D_0 = \{\omega \in \Omega_2 : D_1(\omega) = 0\}$  and  $D_1 = \Omega_2 \setminus \Omega_0$ . For each  $\omega \in \Omega_2$  we consider the homogeneous Fredholm's equation of the second type in  $L_2(\Omega_1)$ :

$$\varphi(x) - (K_\omega \varphi)(x) = \theta. \quad (10)$$

By the Fredholm's theorem for all  $\omega \in D_1$  the equation (10) has only a trivial solution  $\varphi(x) = \varphi_\omega(x) = \theta$  and for all  $\omega \in D_0$  the equation (10) has not trivial solution in  $L_2(\Omega_1)$ . For  $\omega \in D_0$  solution of the equation (10) has a form

$$\varphi(x) = \varphi_\omega(x) = \alpha_1 d_1 \varphi_1(x) + \alpha_2 d_2 \varphi_2(x),$$

where  $d_1(\omega), d_2(\omega) \in \mathbb{C}$  are arbitrary numbers.

Put

$$h(x, y) = (e(y) - D_1(y)) \alpha_1 b_1(y) \varphi_1(x) + (e(y) - D_1(y)) \alpha_2 b_2(y) \varphi_2(x). \quad (11)$$

It is easy to verify that

$$h(x, y) = \begin{cases} \theta, & \text{if } \omega \in D_1, \\ \alpha_1 b_1(\omega) \varphi_1(x) + \alpha_2 b_2(\omega) \varphi_2(x), & \text{if } \omega \in D_0. \end{cases}$$

Hence,  $h(x, \omega) \in L_2(\Omega_1)$  is a solution of (10). It is easy to see that  $h(x, y) \in L_2(\Omega_1 \times \Omega_2)$  and function  $h(x, y)$  (11) is solution of the HPIE (3).



### Conclutions

In this paper, it has been investigated the solvability of the partial integral equation with degenerate kernel given with  $f - T_1 f = g$ , found the exact form of the solution of the partial integral equation with degenerate kernel and proved its uniqueness.

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