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СОДЕРЖАНИЕ

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$(m,\psi,\delta)-$ REGULARITY OF COMPACTS IN $\mathbb{C}^n$

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RESUME

It is known that, the $m-$subharmonic measure $\omega^*(z,E,D)$ of a set $E\subset D,$ related to a domain $D\subset\mathbb{C}^n$, is defined by $m-$subharmonic functions in D. This work is devoted to properties of a weighted $m-$subharmonic measure $\omega^*(z,E,D,m,\psi,\delta)$, in particular, $(m,\psi,\delta)-$regularity of a compact set $K\subset D\subset\mathbb{C}^n$. We generalize the $m-$subharmonic measure and prove that several theorems established in [1] regarding the regularity of the compact set K also hold in the generalized case.

Key words: $m-$subharmonic function, $m-$subharmonic measure, $m-$polar set, globally $m-$regular compact, locally $m-$regular compact.

Introduction. Plurisubharmonic measure and Green function are fundamental concepts in the theory of plurisubharmonic functions. Their applications have provided solutions to many problems in multidimensional complex analysis in a series of fundamental works by A. Sadullaev [4], [5], [6], E. Bedford, A. Taylor [7], J. Siciak [8], V. P. Zaharjuta [9] and others. Further, weighted Green functions and delta-extremal functions, i.e. generalized Green functions are studied in works [10], [11], [12], [13].

One of the important part of the potential theory is theory of $m-$subharmonic (sh_m) functions. It expands and develops the pluripotential theory, which is the main subject for studying analytic functions of several complex variables and plurisubharmonic functions.

The sh_m functions are defined by the operators

$$(dd^cu)^k\wedge\beta^{n-k},\qquad 1\le k\le n,\tag{1}$$

where $d=\partial+\bar\partial,\ d^c=\frac{\partial-\bar\partial}{4i}$ and $\beta=dd^c\left|z\right|^2=\frac{i}{2}\sum_{i=1}^n dz_i\wedge d\bar z_i$ is standard canonical (1,1) form in $\mathbb{C}^n$. Then $dV_n=\frac{1}{n!}\beta^n$ is volume form in $\mathbb{C}^n$. The operator (1) gives the Laplase operator for $k=1$ and the Monge-Ampere operator for $k=n$. The operator (1) is called the complex Hessians operator, as it can be shown that $(dd^cu)^k\wedge\beta^{n-k}=k!\,(n-k)!H_k\left(u\right)\beta^n$, where $u\in C^2\left(D\right)$ and $H_k\left(u\right)=\sum_{1\le j_1<j_2<...<j_k\le n}\lambda_{j_1}\cdot\lambda_{j_2}\cdot\ldots\cdot\lambda_{j_k}-$ is the Hessian of dimension k of the eigenvalue vector $\lambda=(\lambda_1,\lambda_2,...,\lambda_n)$ of the matrix $\left(u_{j,\bar l}\right),\ u_{j,\bar l}=\frac{\partial^2u}{\partial z_j\partial\bar z_l},\ j,l=1,2,...,n.$ The theory related to the class of sh_m functions was constructed in the works [2], [3], [4]. In their studies, the sh_m functions in the class of integrable functions $L^1_{loc}\left(D\right)$ were defined as following.

Definition 1. Let $u\in C^2\left(D\right)$, where $D\subset\mathbb{C}^n$, is called $m-$subharmonic $(1\le m\le n)$ at the point $z^0\in D$, if the eigenvalues $\lambda\left(u\right)=\left(\lambda_1\left(u\right),\,\lambda_2\left(u\right),...,\lambda_n\left(u\right)\right)$ of the matrix $\left.\left(u_{j\bar k}\right)\right|_{z=z^0}$ belong to $\Gamma_{n-m+1}=\{\lambda: H_1\left(\lambda\right)\ge0,\ H_2\left(\lambda\right)\ge0,\ ...,\ H_{n-m+1}\left(\lambda\right)\ge0\,\}$. A function $u\in C^2\left(D\right)$ is called $m-$subharmonic in D if it is $m-$subharmonic at every point of $z^0\in D$.

In other words, a function $u\in C^2\left(D\right)$ is called $m-$subharmonic if the conditions $\left(dd^cu\right)^k\wedge\beta^{n-k}\ge0\,,\ \forall k=1,\,2,...,\,n-m+1$ holds.

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It is known that for all twice differentiable $m-$subharmonic functions $u,\ v_1,\ v_2,\ ...,\ v_{n-m}$ it is true

$$dd^c u\wedge dd^c v_1\wedge dd^c v_2\wedge ...\wedge dd^c v_{n-m}\wedge\beta^{m-1}\geq 0. \tag{2}$$

Moreover, if a twice differentiable function u satisfies (2) for all twice differentiable $m-$subharmonic functions $v_1,\ v_2,\ ...,\ v_{n-m}$ then u is necessarily $m-$subharmonic function. Using this, we can define $m-$subharmonic functions in the class of the upper semicontinuous functions.

Definition 2. A function u is called $m-$subharmonic in the domain $D\subset\mathbb{C}^n$, if it is upper semicontinuous and for any twice differentiable $m-$subharmonic functions $v_1,\ v_2,\ ...,\ v_{n-m}$ the current $dd^c u\wedge dd^c v_1\wedge dd^c v_2\wedge ...\wedge dd^c v_{n-m}\wedge\beta^{m-1}$ defined as

$$\left[dd^c u\wedge dd^c v_1\wedge dd^c v_2\wedge ...\wedge dd^c v_{n-m}\wedge\beta^{m-1}\right](\omega)=$$

$=\int u\wedge dd^c v_1\wedge dd^c v_2\wedge ...\wedge dd^c v_{n-m}\wedge\beta^{m-1}\wedge dd^c\omega,\quad \omega\in\ F^{0,0}$ is positive, i.e.

$$\int u\wedge dd^c v_1\wedge dd^c v_2\wedge ...\wedge dd^c v_{n-m}\wedge\beta^{m-1}\wedge dd^c\omega\geq 0,\quad \forall\omega\geq 0.$$

Class of $m-$subharmonic functions we denote as $sh_m(D)$. It is clear, that

$$psh=sh_1\subset sh_2\subset sh_m\subset ...\subset sh_n=sh. \tag{3}$$

(3) follows, that if $u(z)\in sh_m(D),\ \ u(z)\not\equiv-\infty$, then $u(z)\in L^1_{loc}(D)$.

Definition 3. A set $E\subset D$ is called $m-$polar in $D\subset\mathbb{C}^n$ if there exist a function $u(z)\in sh_m(D),\ u(z)\not\equiv-\infty$, such that $u|_E=-\infty$.

(3) follows also, that a $m-$polar set is polar in the sense of the classical potential theory, so that for $m-$polar set $E\subset D$ the Hausdorf measure $H_{2n-2+0}(E)=0$.

Definition 4. A domain $D\subset\mathbb{C}^n$ is called $m-$regular if there exists a $m-$subharmonic function $\rho(z)$ in D such that $\rho|_D<0,\ \ \lim\limits_{z\to D}\rho(z)=0$, i.e. $D=\{z\in\mathbb{C}^n:\rho(z)<0\}$.

The $m-$subharmonic measure is defined as an extremal function in the class of $m-$subharmonic (sh_m) functions. Let $E\subset D$ be some subset of the domain $D\subset\mathbb{C}^n$. For the sake of simplicity, we assume that D is a bounded and $m-$regular domain, We denote by $\mathcal{U}(E,D)$ the class of all functions $u\in sh_m(D)$, such that $u|_E\leq-1,\ \ u|_D<0$ and let

$$\omega(z,E,D)=\sup\{u(z):\ u(z)\in\mathcal{U}(E,D)\}.$$

Definition 5. The regularization

$\omega^*(z,\ E,\ D)=\overline{\lim\limits_{w\to z}}\,\omega(w,\ E,\ D)=\lim\limits_{\varepsilon\to 0}\sup\limits_{w\in B(z,\varepsilon)}\omega(w,\ E,\ D)$ is called the $m-$subharmonic measure ($\mathcal{P}_m-$measure) of E with respect to D (see [2], [4]).

Let $D\subset\mathbb{C}^n$ be a domain and $K\subset D$ a compact.

Definition 6. A point $z^0\in K$ is said to be globally $m-$regular if$\omega^*(z^0,\ K,D)=-1$. It is said to be locally $m-$regular if for any neighborhood $B,\ z^0\in B\subset\mathbb{C}^n$, the intersection $K\bigcap\bar{B}$ is globally $m-$regular at the point z^0, i.e. $\omega^*\left(z^0,\ K\bigcap\overline{B},\ D\right)=-1$. If all points of a compact set K are globally (or locally) $m-$regular, then the compact set K is called a globally (or locally) $m-$regular compact. (see [2], [4]).

The following Hartogs and Choquet's lemmas are used repeatedly below.

Lemma 1. (Hartogs', see [4]). Suppose that $g(z)$ is a continuous real valued function in a domain $D \subset \mathbb{C}^n$ and $u_j(z)$, $j \in \mathrm{N}$, is a sequence of locally uniformly upper bounded subharmonic functions such that

$$\overline{\lim_{j\to\infty}} u_j(z) \le g(z)$$

at each point $z \in D$. Then for any compact set $K \subset D$, for any $\varepsilon > 0$ there exists an integer j_0 such that $u_j(z) \le g(z) + \varepsilon$, for each $z \in K$ and each $j > j_0$.

Lemma 2. (Choquet technical lemma). For any family $\{u_\alpha(x)\}$, $\alpha \in \Lambda$, of functions of $x \in D \subset \mathbb{R}^n$, there is a countable set $\Lambda_0 \subset \Lambda$such that if we denote $u(x) = \sup\limits_{\alpha\in\Lambda} u_\alpha(x)$ and $v(x) = \sup\limits_{\alpha\in\Lambda_0} u_\alpha(x)$, then $\{x \in D : u(x) < u^*(x)\} \subset \{x \in D : v(x) < v^*(x)\}$ and $u^*(x) \equiv v^*(x)$, where u^*, $v^*-$are regularizations.

Let u_α be a family of upper semicontinuous functions in $D \subset \mathbb{C}^n$ which is locally uniformly bounded from above. Then the upper envelope $u(z) = \sup\limits_{\alpha} u_\alpha(z)$is not always upper semicontinuous. But if we consider the upper semicontinuous regularization $u^*(z) = \lim\limits_{\varepsilon\to 0} \sup\limits_{w\in B(z,\varepsilon)} u(w)$, where $B(z, \varepsilon) \subset D$ is a ball, then u^* is upper semicontinuous and holds

Theorem 1.(see [4]) Let $\{u_\alpha(z)\}$, $\alpha \in \Lambda$, be an arbitrary locally uniformly upper bounded family of $m-$subharmonic functions in the domain $D \subset \mathbb{C}^n$ and $u(z) = \sup\limits_{\alpha}\{u_\alpha(z)\}$. Then the regularization $u^*(z)$ of $u(z)$ is a $m-$subharmonic function in D.

1. (m, ψ, δ)**- subharmonic measure and its properties.** Let $D \subset \mathbb{C}^n$ be a $m-$regular domain, $E \subset D$ be any fixed set and $\psi(z)$ be bounded function in E. We denote by $\mathcal{U}(E, D, \psi, \delta)$ the class of all functions $u(z) \in sh_m(D)$, such that $u|_E \le \psi|_E$, $u|_D < \delta$, where $\delta \in \mathrm{R}$. Using this family of functions, we define the function

$$\omega(z, E, D, \psi, \delta) = \sup\{u(z) : u(z) \in \mathcal{U}(E, D, \psi, \delta)\}.$$

Definition 1.1. The function

$$\omega^*(z, E, D, \psi, \delta) = \overline{\lim_{w\to z}} \omega(w, E, D, \psi, \delta)$$

is called a $(m, \psi, \delta)-$subharmonic measure ($_{(m,\psi,\delta)}-$measure) of the set E with respect to D.

Note that $\omega^*(z, E, D, -1, 0)$, $\psi \equiv -1$, $\delta = 0$, coincides with the $m-$ measure of the potential theory in the class of $u(z) \in sh_m(D)$, i.e. $\omega^*(z, E, D, -1, 0) = \omega^*(z, E, D)$. The weighted $_{(m,\psi)}-$ measure $\omega^*(z, E, D, \psi, 0)$, case $\delta = 0$, was considered in our previous work [15].

As can be seen from the definition 1.1., the inequality $\inf\limits_{z\in E}\psi(z) \le \omega^*(z, E, D, \psi, \delta) \le \delta$ holds for all $z \in D$. By Theorem 0.1., the function $\omega^*(z, E, D, \psi, \delta)$ is $m-$subharmonic in D. If $\delta \le \inf\limits_{z\in E}\psi(z)$, then $\omega^*(z, E, D, \psi, \delta) = \delta$, $\forall z \in D$. Therefore, in this paper, we will consider the special case where $\delta > \sup\limits_{z\in E}\psi(z)$ is satisfied.

In this context, we present several characteristics of the $(m, \psi, \delta)-$subharmonic measure;

Proposition 1.1. a) let $E_1 \subset E_2 \subset D_1 \subset D_2$. Then

$\omega^*(z, E_2, D_2, \psi, \delta) \le \omega^*(z, E_1, D_2, \psi, \delta) \le \omega^*(z, E_1, D_1, \psi, \delta)$ for all $z \in D_1$.

b) let $\psi_1|_E \le \psi_2|_E$. Then $\omega^*(z, E, D, \psi_1, \delta) \le \omega^*(z, E, D, \psi_2, \delta)$ for all $z \in D$.

c) let $\sup\limits_{z\in E}\psi(z) < \delta_1 \le \delta_2, \delta_1, \delta_2 \in \mathrm{R}$. Then $\omega^*(z, E, D, \psi, \delta_1) \le \omega^*(z, E, D, \psi, \delta_2), \forall z \in D$.

d) let $c \ge 1$. Then $\omega^*\left(z, E, D, \frac{\psi}{c}, \delta\right) = \frac{1}{c}\omega^*(z, E, D, \psi, c\delta), \forall z \in D$.

e) let $c \le 0$. Then

$$\omega^*(z, E, D, \psi + c, \delta) = c + \omega^*(z, E, D, \psi, \delta - c), \forall z \in D.$$

The proofs of Proposition 1.1. follow easily from the definition of the (m, ψ, δ) −subharmonic measure.

Proposition 1.2 (On two constants Theorem). If the function $u(z)$ is $m-$subharmonic in the domain $D \subset \mathbb{C}^n$ and $u|_D < C$, $u|_E \le c$, where $E \subset D$, $c < C$ then the inequality

$$u(z) \le C \cdot \left(1 + \frac{\omega^*(z, E, D, \psi, \delta) - \delta}{\delta - \inf\limits_{z\in E}\psi(z)}\right) - c \cdot \frac{\omega^*(z, E, D, \psi, \delta) - \delta}{\delta - \inf\limits_{z\in E}\psi(z)}$$

holds for all $z \in D$.

The proof of the proposition 1.2. follows easily from the relation

$$\frac{u(z) - C}{C - c}\left(\delta - \inf_{z\in E}\psi(z)\right) + \delta \in \mathcal{U}(E, D, \psi, \delta).$$

Proposition 1.3. The inequality

$$\left(\delta - \inf_{z\in E}\psi(z)\right) \cdot \omega^*(z, E, D) + \delta \le \omega^*(z, E, D, \psi, \delta) \le \left(\delta - \sup_{z\in E}\psi(z)\right) \cdot \omega^*(z, E, D) + \delta$$

holds for any set $E \subset D$ and for all $z \in D$.

It follows from proposition 1.3 that the measure $\omega^*(z, E, D, \psi, \delta)$ is either nowhere δ or identically δ. The latter holds if and only if E is $m-$polar in D.

Proof of proposition 1.3. Take an arbitrary function $u(z) \in \mathcal{U}(E, D)$ i.e. $u(z)|_E \le -1$, $u(z)|_D < 0$. Then $\left(\delta - \inf\limits_{z\in E}\psi(z)\right) \cdot u(z) + \delta \in sh_m(D)$. From $\psi(z)|_E < \delta$, it follows $\delta - \inf\limits_{z\in E}\psi(z) > 0$. Note that

$$\left(\left(\delta - \inf_{z\in E}\psi(z)\right) \cdot u(z) + \delta\right)\Big|_D \le \delta, \quad \left(\left(\delta - \inf_{z\in E}\psi(z)\right) \cdot u(z) + \delta\right)\Big|_E \le \inf_{z\in E}\psi(z) - \delta + \delta \le \psi|_E.$$

Consequently $\left(\delta - \inf\limits_{z\in E}\psi(z)\right) \cdot u(z) + \delta \in \mathcal{U}(E, D, \psi, \delta)$ and $\left(\delta - \inf\limits_{z\in E}\psi(z)\right) \cdot u(z) + \delta \le \omega^*(z, E, D, \psi, \delta)$.

As the function u is arbitrary, we get the inequality

$\left(\delta - \inf\limits_{z\in E}\psi(z)\right) \cdot \omega^*(z, E, D) + \delta \le \omega^*(z, E, D, \psi, \delta)$ for all $z \in D$.

Now we show that the inequality $\omega^*(z, E, D, \psi, \delta) \leq \left(\delta - \sup\limits_{z\in E}\psi(z)\right) \cdot \omega^*(z, E, D) + \delta$ holds. Take any function $u(z) \in \mathcal{U}(E, D, \psi, \delta)$ and consider the function $\frac{u(z)-\delta}{\delta-\sup\limits_{z\in E}\psi(z)}$. It can be easily verify that the function $\frac{u(z)-\delta}{\delta-\sup\limits_{z\in E}\psi(z)}$ is $m-$subharmonic in D and satisfies the following conditions:

$\left.\frac{u(z)-\delta}{\delta-\sup\limits_{z\in E}\psi(z)}\right|_D < 0$ and $\left.\frac{u(z)-\delta}{\delta-\sup\limits_{z\in E}\psi(z)}\right|_E < -1.$

Thus $\frac{u(z)-\delta}{\delta-\sup\limits_{z\in E}\psi(z)} \in \mathcal{U}(E, D)$ and $\frac{u(z)-\delta}{\delta-\sup\limits_{z\in E}\psi(z)} \leq \omega^*(z, E, D)$. Therefore the inequality $\omega^*(z, E, D, \psi, \delta) \leq \left(\delta - \sup\limits_{z\in E}\psi(z)\right) \cdot \omega^*(z, E, D) + \delta$ follows from the arbitrariness of the function $u(z) \in \mathcal{U}(E, D, \psi, \delta)$. *The proposition 1.3. is proven.*

Proposition 1.4. Let $E = \bigcup\limits_{j=1}^{\infty} E_j,\ \ E_j \subset D, \forall j \in \mathrm{N},\ \delta \leq 0$. Then for all $z \in D$ the inequality

$$\omega^*(z, E, D, \psi, \delta) \geq \sum_{j=1}^{\infty} \omega^*(z, E_j, D, \psi, \delta)$$

holds.

Proof. Take $\forall u_j \in \mathcal{U}(E_j, D, \psi, \delta)$ and consider the class

$$\left\{\sum_{j=1}^{\infty} u_j(z) : \quad u_j \in \mathcal{U}(E_j, D, \psi, \delta)\right\}.$$

Since u_j is $m-$subharmonic and negative in D, it follows that the sum $\sum_{j=1}^{\infty} u_j(z)$ is also $m-$subharmonic function in D. We can easily check that $\sum_{j=1}^{\infty} u_j(z) \in \mathcal{U}(E, D, \psi, \delta)$.

Hence,

$$\omega(z, E, D, \psi, \delta) \geq \sup\left\{\sum_{j=1}^{\infty} u_j(z) : \quad u_j \in \mathcal{U}(E_j, D, \psi, \delta)\right\} =$$

$$= \sum_{j=1}^{\infty} \sup\{u_j(z) : \quad u_j \in \mathcal{U}(E_j, D, \psi, \delta)\} = \sum_{j=1}^{\infty} \omega(E_j, D, \psi, \delta).$$

Now we investigate the sets$P_j = \{z \in D : \quad \omega(z, E_j, D, \psi, \delta) < \omega^*(z, E_j, D, \psi, \delta)\},\ j \in \mathrm{N}$. We know that the sets P_j are $m-$polar, and their Lebesgue measure is zero. Therefore, the Lebesgue measure of $P = \bigcup_{j=1}^{\infty} P_j$ is also zero, i.e. $mes(P) = mes\left(\bigcup_{j=1}^{\infty} P_j\right) = 0$. If we take an upper regularization, we see

$$\omega^*(z, E, D, \psi, \delta) \geq \overline{\lim_{w\to z}} \sum_{j=1}^{\infty} \omega(z, E_j, D, \psi, \delta) \geq$$

$$\geq \overline{\lim_{w\to z,\, w\in D\setminus P}} \sum_{j=1}^{\infty} \omega^*(z, E_j, D, \psi, \delta) = \sum_{j=1}^{\infty} \omega^*(z, E_j, D, \psi, \delta).$$

The Proposition 1.4. is proven.

Proposition 1.5. If $E \subset\subset D$, then $\lim\limits_{z\to\partial D}\omega^*(z,\, E,\, D,\, \psi,\delta) = \delta$.

Proof. Since D is a $m-$regular domain, there exists a function such that $\rho\,(z) \in sh_m\,(D)$ and $\lim\limits_{z\to\partial D}\rho\,(z) = 0$. It followed that $C\cdot p\,(z) + \delta \in \mathcal{U}\,(E,\, D,\, \psi,\delta)\,$, where $C = \frac{\inf\limits_{z\in E}\psi(z)-\delta}{\max\limits_{z\in E}p(z)}$. So, from the relations $C\cdot\rho+\delta \le \omega^*(z,\, E,\, D,\, \psi,\delta) \le \delta$ and $\lim\limits_{z\to\partial D} C\cdot\rho = 0$, we get $\lim\limits_{z\to\partial D}\omega^*(z,\, E,\, D,\psi,\delta) = \delta$. *The proposition 1.5. is proven.*

Proposition 1.6. Let $E \subset\subset D_1$ and $D_j \subset D_{j+1},\ \bigcup_{j=1}^{\infty} D_j = D,\ j \in \mathrm{N}$. Then

$$\lim_{j\to\infty}\omega^*\,(z,\, E,\, D_j,\, \psi,\delta) = \omega^*\,(z,\, E,\, D,\, \psi,\delta)\,.$$

Proof. According to the proposition 1.1., the inequality $\omega^*\,(z,\, E,\, D_j,\, \psi,\delta) \ge \omega^*\,(z,\, E,\, D_{j+1},\, \psi,\delta)$ is valid. It follows that $\omega^*\,(z,\, E,\, D_j,\, \psi,\delta)$ is decreasing with respect to j and $\lim\limits_{j\to\infty}\omega^*\,(z,\, E,\, D_j,\, \psi,\delta) = \omega\,(z) \in sh_m\,(D)$. Therefore the inequality $\lim\limits_{j\to\infty}\omega^*\,(z,\, E,\, D_j,\, \psi,\delta) \ge \omega^*\,(z,\, E,\, D,\, \psi,\delta)$ holds for all $z \in D$.

Now we have to show that $\lim\limits_{j\to\infty}\omega^*\,(z,\, E,\, D_j,\, \psi,\delta) \le \omega^*\,(z,\, E,\, D,\, \psi,\delta)\,$. We choose $\varepsilon_j > 0$, $\lim\limits_{j\to\infty}\varepsilon_j = 0$, so that the following relationships $\{z \in D :\, \rho \le -\varepsilon_j\} \subset D_j$ hold. We note, that $\rho \in sh_m\,(D)\,,\ \lim\limits_{z\to\partial D}\rho\,(z) = 0$. Take an arbitrary $u \in \mathcal{U}\,(E,\, D_j,\, \psi,\delta)$ and consider the function

$$v(z) = \begin{cases} \max\{u - C\varepsilon_j,\, C\rho+\delta\},\ z \in D_j \\ C\rho+\delta,\ z \in D\backslash D_j \end{cases},$$

where $C = \frac{\inf\limits_{z\in E}\psi(z)-\delta}{\max\limits_{z\in E}p(z)}$. It is not difficult to see that $v \in \mathcal{U}\,(E,\, D,\, \psi,\delta)$ and $u\,(z) - C\varepsilon_j \le v(z) \le \omega^*\,(z,\, E,\, D,\, \psi,\delta)$ for all $z \in D_j$. Since $u \in \mathcal{U}\,(E,\, D_j,\, \psi,\delta)$ is arbitrary, the inequality $\omega^*\,(z,\, E,\, D_j,\, \psi,\delta) - C\varepsilon_j \le \omega^*\,(z,\, E,\, D,\, \psi,\delta)$ holds for arbitrary $z \in D_j$. As a result, we obtain the inequality $\lim\limits_{j\to\infty}\omega^*\,(z,\, E,\, D_j,\, \psi,\delta) \le \omega^*\,(z,\, E,\, D,\, \psi,\delta)$ for all $z \in D$. *The proof is over.*

Proposition 1.7. a) let $E \subset D$ be an arbitrary set and a function $\psi\,(z)$ be a lower semicontinuous in $V \subset D$, where V some neighborhoods of E. Then there exists a sequence of open sets $U_j \supset E,\ \ U_j \supset U_{j+1}$ such that

$$\left(\lim_{j\to\infty}\omega^*\,(z,\, U_j,\, D,\, \psi,\delta)\right)^* = \omega^*\,(z,\, E,\, D,\, \psi,\delta)\,.$$

b) let $U \subset D$ be an open set and $U = \bigcup_{j=1}^{\infty} K_j$, where $K_j \subset K_{j+1}^0$ are a compact sets and $\psi\,(z)$ is an upper semicontinuous function in U. Then

$$\omega^*\,(z,\, K_j,\, D,\, \psi) \downarrow \omega^*\,(z,\, U,\, D,\, \psi)\,.$$

Proof. a) By technical Choquet Lemma (see [4]) there exists a class of countable functions $\{u_k\} \subset \mathcal{U}\,(E,\, D,\, \psi,\delta)$ such that $\left(\sup\limits_k u_k\,(z)\right)^* = \omega^*\,(z,\, E,\, D,\, \psi,\delta)\,$. It is evident that the functional sequence $v_j\,(z) = \max\{u_1\,(z)\,,\ u_2\,(z)\,,\ \ldots\,,\ u_j\,(z)\}$ is increasing and

$\left(\lim\limits_{j\to\infty} v_j(z)\right)^* = \omega^*(z,\, E,\, D,\, \psi, \delta)$. Now take the sets $U_j = \left\{z \in V : v_j < \psi(z) + \frac{1}{j}\right\}$, $j \in \mathrm{N}$. The sets U_j are open because the functions $v_j(z) - \psi(z)$ are upper semicontinuous in V. It is easy to see, that $U_j \supset U_{j+1}$ and $v_j - \frac{1}{j} \in \mathcal{U}(U_j,\, D,\, \psi, \delta)$ for all $j \in \mathrm{N}$. By proposition 1.1., we get $\omega^*(z, U_j,\, D,\, \psi, \delta) \le \omega^*(z,\, E,\, D,\, \psi, \delta)$, $\forall j \in \mathrm{N}$. On the other hand $v_j - \frac{1}{j} \le \omega(z,\, U_j,\, D,\, \psi, \delta)$, $\forall j \in N$. As a result,

$$v_j - \frac{1}{j} \le \omega(z,\, U_j,\, D,\, \psi, \delta) \le \omega^*(z,\, E,\, D,\, \psi, \delta),\ \ \forall j \in N.$$

Now we take the limit $j \to \infty$ and the regularization:

$$\left\{\lim_{j\to\infty}\left(v_j(z) - \frac{1}{j}\right)\right\}^* \le \left\{\lim_{j\to\infty} \omega(z,\, U_j,\, D,\, \psi, \delta)\right\}^* \le \left\{\lim_{j\to\infty} \omega(z,\, E,\, D,\, \psi, \delta)\right\}^*.$$

Consequently $\left\{\lim\limits_{j\to\infty} \omega(z,\, U_j,\, D,\, \psi, \delta)\right\}^* = \omega^*(z,\, E,\, D,\, \psi, \delta)$. The proof of a) is over.

b). By proposition 1.1. we have $\omega^*(z,\, K_j,\, D,\, \psi, \delta) \ge \omega^*(z,\, K_{j+1},\, D,\, \psi, \delta)$. The functions $\omega^*(z,\, K_j,\, D,\, \psi, \delta)$ are decreasing $m-$subharmonic, and the limit is $m-$subharmonic, i.e.

$$\lim_{j\to\infty} \omega^*(z,\, K_j,\, D,\, \psi, \delta) = \omega(z),\ \omega(z) \in sh_m(D).$$

By the monotony $\omega^*(z,\, K_j, D, \psi, \delta) \ge \omega^*(z,\, U,\, D, \psi, \delta)$, $\forall j \in \mathbb{N}$ and $\omega(z) \ge \omega^*(z,\, U,\, D,\, \psi, \delta)$ for all $z \in D$. Now we have to show that $\omega(z) \le \omega^*(z,\, U,\, D,\, \psi, \delta)$. Since the function $\psi(z)$ is an upper semicontinuous in U,

$$\forall z \in \bigcup_{j=1}^{\infty} K_j = U,\ \exists j_0,\, \forall j > j_0,\ \omega^*(z, K_j, D, \psi, \delta) \le \psi(z).$$

It follows from this that $\omega(z)|_U \le \psi(z)|_U$. Therefore $\omega(z) \in \mathcal{U}(U, D, \psi, \delta)$ and $\omega(z) \le \omega^*(z, U,\, D,\, \psi, \delta)$. *The proof is over.*

2. $(m,\, \psi, \delta)$ **−regularity of compacts.** Let the function $\psi(z)$ be extended to the domain D as a function from the class $\mathcal{U}(E, D, \psi, \delta)$ i.e. if there is a function

$$\tilde{\psi} \in sh_m(D),\ \tilde{\psi}\Big|_E = \psi|_E,\ \ \tilde{\psi}\Big|_D < \delta, \tag{2.1}$$

then it is ibvious $\omega(z, E,\, D,\, \psi, \delta) \ge \tilde{\psi}(z)$, $\forall z \in D$ and

$$\omega(z, E,\, D,\, \psi, \delta) = \psi(z),\, \forall z \in E. \tag{2.2}$$

However, if condition (2.1) is not satisfied, then, in general, equality (2.2) does not hold. In the following example, it can be observed that equality (2.2) fails to hold.

Example 2.1. Let $\psi(z) = 1 - |z|^2$, $\delta = 2$, $D = B(0,2) \subset \mathbb{C}^n$, $E = \bar{B}(0,1) \subset \mathbb{C}^n$. According to the maximum principle and by the definition 1.1.,

$$\omega\left(z,\, \bar{B}(0,1), B(0,2), 1 - |z|^2, 2\right) = 0 \ne \psi(z),\ \forall z \in B(0,1).$$

We assume that condition (2.2) holds in the definitions of $\omega(z, E, D, \psi, \delta)$. We also assume, that $D \subset \mathbb{C}^n$ is a $m-$regular domain and $K \subset D$ is a compact.

Definition 2.1. A point $z^0 \in K$ is said to be globally $(m, \psi, \delta)-$regular if$\omega^*(z^0, K, D, \psi, \delta) = \psi(z^0)$. It is said to be locally $(m, \psi, \delta)-$regular if for any neighborhood B, $z^0 \in B \subset \mathrm{C}^n$, the intersection $K \bigcap \bar{B}$ is globally $(m, \psi, \delta)-$regular at the point z^0, i.e. $\omega^*\left(z^0, K \bigcap \overline{B}, D, \psi, \delta\right) = \psi(z^0)$. If all points of a compact set K are globally (or locally) $(m, \psi, \delta)-$regular, then the compact set K is called a globally (or locally)$(m, \psi, \delta)-$regular compact.

Proposition 2.1. Let $\delta_1, \delta_2 \in \mathrm{R}$, $\delta_1 \leq \delta_2$ and K be a compact set in $D \subset \mathbb{C}^n$ and condition (2.2) be satisfied, i.e. $\omega(z, E, D, \psi, \delta_1) = \psi(z), \forall z \in E$. If K is $(m, \psi, \delta_2)-$regular at $z_0 \in K$, then K is $(m, \psi, \delta_1)-$regular at $z_0 \in K$. Thusly, if K is $(m, \psi, \delta_2)-$regular at $z_0 \in K$, then K is $(m, \psi, \delta_1)-$regular at $z_0 \in K$, $\delta_1 \leq \delta_2$.

The proof of proposition 2.1. easily follows from definition 2.1. and proposition 1.1.

Theorem 2.1. Let K be $(m, \psi, \delta)-$regular a compact set and $\psi(z)$ be a continuous in the compact set K. Then $\omega^*(z, K, D, \psi) \equiv \omega(z, K, D, \psi) \in C\left(\bar{D}\right)$ for any $z \in D$.

Proof. Let K be $(m, \psi, \delta)-$regular a compact i.e. $\omega^*(z, K, D, \psi, \delta)|_K = \psi|_K$. It is evident that $\omega^*(z, K, D, \psi, \delta) \subset \mathcal{U}(K, D, \psi, \delta)$ and $\omega^*(z, K, D, \psi, \delta) \equiv \omega(z, K, D, \psi, \delta)$.

Now we prove that $\omega^*(z, K, D, \psi, \delta)$ is continuous in $\bar{D}$. Let us fix $\varepsilon > 0$ to be sufficiently small and then construct the domain $G_\varepsilon = \{z \in D : \omega^*(z, K, D, \psi, \delta) < \delta - \varepsilon\}$, where $\delta - \varepsilon > \psi|_K$. It is easy to see that according to the proposition 1.5., the relation $K \subset G_\varepsilon \subset\subset D$ is valid. There exists a sequence of monotonic functions $u_j(z) \in sh_m(G) \bigcap C^\infty(G)$ such that $u_j(z) \downarrow \omega^*(z, K, D, \psi, \delta)$ holds for any $z \in G$, where $G_\varepsilon \subset\subset G \subset\subset D$. By applying Hartog's lemma to the sets G and $\bar{G}_\varepsilon$, we establish the relation

$$\exists j_1 \in \mathrm{N}, \ \ \forall j > j_1, \ \ \forall z \in \bar{G}_\varepsilon : \ \ u_j(z) < \delta.$$

On the other hand, since the function $\psi(z)$ is continuous on the compact set K, according to Whitney's theorem [14], there exists some continuous function $\tilde{\psi}(z)$ in D such that $\left.\tilde{\psi}(z)\right|_K = \psi(z)|_K$. Now we consider open sets $U_\varepsilon = \left\{z \in D : \omega^*(z, K, D, \psi, \delta) < \tilde{\psi}(z) + \varepsilon\right\}$. It is clear that $K \subset U_\varepsilon$. We again apply Hartogs' lemma to the pair of sets U_ε and K and get

$$\exists j_2 \in \mathrm{N}, \forall j > j_2, \forall z \in K : \ u_j(z) < \psi(z) + 2\varepsilon.$$

Let us consider the function

$$v(z) = \begin{cases} \max\{u_j(z) - 2\varepsilon, \ \omega^*(z, K, D, \psi, \delta)\}, & z \in G_\varepsilon \\ \omega^*(z, K, D, \psi, \delta), & z \in D \backslash G_\varepsilon \end{cases}.$$

It is obvious that $v|_K \leq \psi|_K$, $v|_D < \delta$ for $\forall j > j_3 = \max\{j_1, j_2\}$. It implys $v(z) \in \mathcal{U}(K, D, \psi, \delta)$ and $v(z) \leq \omega^*(z, K, D, \psi, \delta)$. Consequently

$$u_j(z) - 2\varepsilon \leq \omega^*(z, K, D, \psi, \delta) \leq u_j(z), \forall j > j_3, \forall z \in G_\varepsilon.$$

Therefore, $\omega^*(z, K, D, \psi, \delta)$ is the uniform limit of the $u_j(z)$ in G_ε. This implys that $\omega^*(z, K, D, \psi, \delta) \in C(G_\varepsilon)$. Since $G_\varepsilon \subset\subset G \subset\subset D$, and $\varepsilon > 0$ is arbitrary, then $\omega^*(z, K, D, \psi, \delta) \in C(D)$. *The theorem is proven.*

Theorem 2.2. Let $\psi \in C(K)$ and condition (2.2) be satisfied, i.e. $\omega(z, E, D, \psi, \delta_1) = \psi(z), \forall z \in K$. A fixed point $z^0 \in K \subset C^n$ is locally (m, ψ, δ) −regular if and only if it is locally m−regular, $\omega^*\left(z^0, K \bigcap \overline{B}, D\right) = -1$.

Proof. To prove this theorem, we show that if the point $z^0 \in K$ is not local m−regular, then it is not local (m, ψ, δ) −regular and conversely, if point $z^0 \in K$ is not local (m, ψ, δ) −regular, then it is not local m−regular. Let us assume that the point $z^0 \in K$ is not a local m−regular. i.e. there exists a ball B such that $z^0 \in B \subset D$ and the equality $\omega^*\left(z^0, K \bigcap \overline{B}, D\right) \geq -1+\varepsilon, 0 < \varepsilon < 1$ is valid. According to monotonicity $\omega^*\left(z^0, K \bigcap \overline{B_1}, D\right) \geq -1+\varepsilon$ for any ball B_1, where $z^0 \in B_1 \subset B$. Therefore by proposition 1.3.

$$\omega^*\left(z^0, K \bigcap \overline{B_1}, D, \psi, \delta\right) \geq \left(\delta - \inf_{z \in K \bigcap \bar{B}_1} \psi(z)\right) \cdot \omega^*\left(z^0, K \bigcap \overline{B_1}, D\right) + \delta \geq$$
$$\geq \left(\delta - \inf_{z \in K \bigcap \bar{B}_1} \psi(z)\right)(-1+\varepsilon) + \delta = \delta \cdot \varepsilon + \inf_{z \in K \bigcap \bar{B}_1} \psi(z) \cdot (1-\varepsilon).$$

Since $\psi(z)$ is continuous, by choosing the neighborhood B_1 and ε small enough, we obtain $\inf_{x \in K \bigcap \bar{B}_1} \psi(z) \geq \frac{\psi(z^0) - \varepsilon \cdot \delta}{1-\varepsilon}$. From this inequality we get the relation

$$\omega^*\left(z^0, K \bigcap \overline{B_1}, D, \psi\right) \geq \varepsilon \cdot \delta + \inf_{z \in K \bigcap \bar{B}_1} \psi(z) \cdot (1-\varepsilon) > \varepsilon \cdot \delta + \frac{(\psi(z^0) - \varepsilon \cdot \delta) \cdot (1-\varepsilon)}{1-\varepsilon} = \psi\left(z^0\right).$$

Therefore $z^0 \in K$ is not local (m, ψ, δ)− regular.

Conversely, we have to show that if the point $z^0 \in K$ is not local (m, ψ, δ) −regular, then it is not local m−regular. Suppose $z^0 \in K$ is not (m, ψ, δ) −regular i.e., there exists the ball $B, z^0 \in B \subset D$, such that $\omega^*\left(z^0, K \bigcap \overline{B}, D, \psi, \delta\right) \geq \psi(z^0) + \alpha, \alpha > 0$. By using the previous technique we get $\omega^*\left(z^0, K \bigcap \overline{B_1}, D, \psi, \delta\right) \geq \psi(z^0) + \alpha$ for any ball B_1, where $z^0 \in B_1 \subset B$. Therefore by proposition 1.3.,

$$\psi\left(z^0\right) + \alpha \leq \omega^*\left(z^0, K \bigcap \overline{B_1}, D, \psi, \delta\right) \leq \left(\delta - \sup_{z \in K \bigcap \overline{B_1}} \psi(z)\right) \cdot \omega^*\left(z^0, K \bigcap \overline{B_1}, D\right) + \delta.$$

Given that $\psi(z)$ is continuous, we can make $\sup_{z \in K \bigcap \bar{B}_1} \psi(z) < \psi(z^0) + \alpha < \delta$ by selecting sufficiently small values for α and the neighborhood B_1. Thus,

$$\psi\left(z^0\right) + \alpha \leq \omega^*\left(z^0, K \bigcap \overline{B_1}, D, \psi, \delta\right) < \left(\delta - \psi\left(z^0\right) - \alpha\right) \cdot \omega^*\left(z^0, K \bigcap \overline{B_1}, D\right) + \delta.$$

From the last inequality we get $\omega^*\left(z^0, K \bigcap \overline{B_1}, D\right) > -1$. Hence $z^0 \in K$ is not the m−regular point. *The theorem is proven.*

Theorem 2.3. Let the function $\psi(z)$ be extended to $\mathcal{U}(K, D, \psi, \delta)$ as a strictly m-subharmonic function in some neighbourhood $D^+ \supset \bar{D}$ of closure $\bar{D}$, i.e, there exists a function $\tilde{\psi}$ that is strictly m-subharmonic in the domain D^+ and $\tilde{\psi}\Big|_K = \psi|_K$, $\tilde{\psi}\Big|_D < \delta$. Then a fixed point $z^0 \in K \subset D$ is locally (m, ψ, δ) −regular if and only if it is globally (m, ψ, δ) −regular.

Here, a function $\psi(z)$ is strictly m-subharmonic function in the domain D, if for any compact domain $G \subset\subset D$, there exists $\varepsilon > 0$: $\psi(z) - \varepsilon |z|^2 \in sh_m(G)$.

Proof. It is clear that a locally (m,ψ,δ) −regular point is also globally (m,ψ,δ) −regular. We will prove the converse: if $z^0 \in K$ is a globally (m,ψ,δ) −regular point, then it is also locally (m,ψ,δ) −regular point. Let us assume that the point $z^0 \in K$ is globally (m,ψ,δ) −regular, which means that $\omega^*(z^0, K, D, \psi, \delta) = \psi(z^0)$. According to the theorem's condition if the function $\psi(z)$ be extend as strictly m-subharmonic function in the domain D^+.Then there exists a constants $\varepsilon > 0$ such that the function $\tilde{\psi}(z) - \varepsilon|z - z^0|^2$ is a strictly m−subharmonic in the domain D, where $\tilde{\psi}$ is a strictly m-subharmonic function in the domain D and $\tilde{\psi}\Big|_K = \psi|_K$, $\tilde{\psi}\Big|_D < \delta$. Now we will fix the function $u(z)$ that satisfies the condition;

$$u(z) \in sh_m(D), \ u|_{K\bigcap \bar{B}_r} \leq -1, \ u|_D < 0,$$

where $B_r = B(z^0, r) \subset\subset D$, $\max\limits_{z\in\bar{B}_r}\tilde{\psi} + r^2 < \delta$. It can be easily seen that the relation $\varepsilon r^2(u(z)+1) - \varepsilon|z-z^0|^2 \leq 0$ is appropriate for any $z \in K$. Thus, the function $\varphi(z) = \varepsilon r^2(u(z)+1) + \tilde{\psi}(z) - \varepsilon|z-z^0|^2$ is m−subharmonic in the domain D and $\varphi|_K \leq \psi|_K$, $\varphi|_D < \delta$. Hence, $\varphi \in \mathcal{U}(K, D, \psi, \delta)$ and

$$\varphi(z) \leq \omega^*(z, K, D, \psi, \delta), \forall z \in D.$$

Thus,

$$\varepsilon r^2\left(\omega^*\left(z, K\bigcap \bar{B}_r, D\right)+1\right) + \tilde{\psi}(z) - \varepsilon\left|z - z^0\right|^2 \leq \omega^*(z, K, D, \psi, \delta), \ \forall z \in D.$$

Putting here $z = z^0$ we have

$$\varepsilon r^2\left(\omega^*\left(z^0, K\bigcap \bar{B}_r, D\right)+1\right) + \psi\left(z^0\right) \leq \omega^*\left(z^0, K, D, \psi, \delta\right) = \psi\left(z^0\right)$$

Hence,

$$\varepsilon r^2\left(\omega^*\left(z^0, K\bigcap \bar{B}_r, D\right)+1\right) \leq 0,$$

which implies $\omega^*\left(z^0, K\bigcap \bar{B}_r, D\right) = -1$.

This implies that the point $z^0 \in K$ is locally m−regular. According to Theorem 2.2. we conculede the local (m,ψ,δ) −regularity of the point $z^0 \in K$. *The theorem is proven.*

From the Theorem 2.2. and the Theorem 2.3., we obtain several important corollaries.

Corollary 2.1. If the compact set $K \subset D$ is globally (m,ψ,δ) −regular, where the function $\psi(z)$ is extended to $\mathcal{U}(K, D, \psi, \delta)$ as a strictly m-subharmonic function in some neighbourhood $D^+ \supset \bar{D}$ of closure $\bar{D}$, then K is locally m−regular.

Corollary 2.2. If ψ_1 and ψ_2 are extended to $\mathcal{U}(K, D, \psi_1, \delta)$ and $\mathcal{U}(K, D, \psi_2, \delta)$ as strictly m-subharmonic functions in some neighbourhood $D^+ \supset \bar{D}$ of closure $\bar{D}$, respectively, then the point $z^0 \in K \subset D$ is (m,ψ_1,δ) −regular if and only if it is (m,ψ_2,δ) −regular.

Corollary 2.3. If the compact set $K \subset D$ is globally (m,ψ,δ) −regular, where $\psi(z)$ is extended to $\mathcal{U}(K, D, \psi, \delta)$ as a strictly m-subharmonic function in some neighbourhood $D^+ \supset \bar{D}$ of closure $\bar{D}$, then K is not m−polar at each of its point. It means that for any $z^0 \in K$ and for any neighborhood $B \subset D$, $z^0 \in B$ the intersection $E = B\bigcap K$ is not m−polar.

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REZYUME

Ma'lumki, $D \subset \mathbb{C}^n$ sohaga tegishli $E \subset D$ to'plamining m-subgarmonik o'lchovi $\omega^*(z, E, D)$ funksiya D sohadagi m-subgarmonik funksiyalar yordamida aniqlanadi. Ushbu ish $\omega^*(z, E, D, m, \psi, \delta)$ vaznli m-subgarmonik o'lchov xossalariga, xususan, $K \subset D \subset \mathbb{C}^n$ kompakt to'plamning (m, ψ, δ)-regulyarligiga bag'ishlangan. Biz m-subgarmonik o'lchovni umumlashtiramiz va [1] maqolada K kompakt to'plamning regulyarligi bo'yicha keltirilgan bir nechta teoremalar umumlashtirilgan holatda ham bajarilishini isbotlaymiz.

Kalit so'zlar: m-subgarmonik funksiya, m-subgarmonik o'lchov, m-polyar to'plam, global m-regulyar kompakt, lokal m-regulyar kompakt.

РЕЗЮМЕ

Известно, что m-субгармоническая мера $\omega^*(z, E, D)$ множества $E \subset D$, связанная с областью $D \subset \mathbb{C}^n$, определяется m-субгармоническими функциями в D. Настоящая работа посвящена изучению свойств m-субгармонической меры с весом $\omega^*(z, E, D, m, \psi, \delta)$, в частности, (m, ψ, δ)-регулярности компактного множества $K \subset D \subset \mathbb{C}^n$. Обобщается понятие m-субгармонической меры и доказывается, что некоторые теоремы, доказанные в [1], относительно регулярности компактного множества K, также верны в обобщенном случае.

Ключевые слова: m-субгармоническая функция, m-субгармоническая мера, m-полярное множество, глобально m-регулярный компакт, локально m-регулярный компакт.