RESEARCH ARTICLE | JUNE 03 2024

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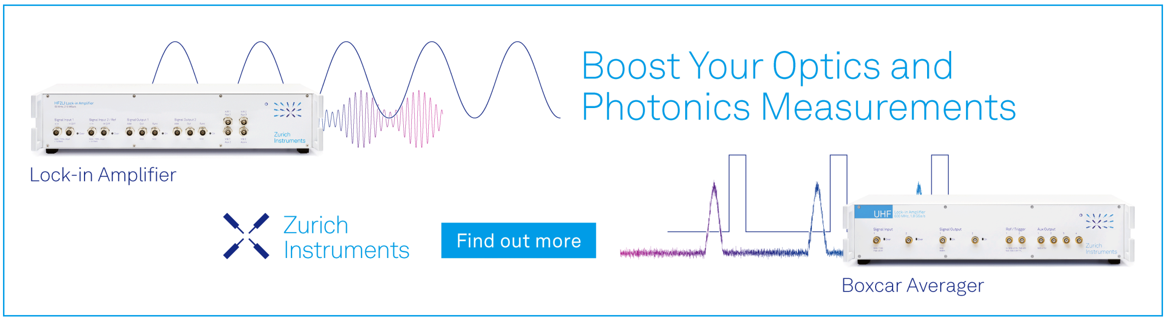
AIP Conf. Proc. 3119, 020005 (2024)
https://doi.org/10.1063/5.0214779



Temperature Field Distribution in Layered Plates

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Abstract. A two-dimensional problem of the stationary distribution of heat in an isotropic plate consisting of a "sandwich"-type three-layer composite material is considered. Temperature heat flux is set at one boundary and convective heat transfer is set at the other boundaries. The solution is obtained in the form of infinite Fourier series. Calculation examples are given for layered materials with different thermal conductivity coefficients and different plate thicknesses. The cases of the presence of a heat-insulating layer are investigated. To justify the effectiveness of the proposed approach, the problem under consideration was solved by simulation based on the Ansys Mechanical APDL 2022 R1 application package.

INTRODUCTION

At present, due to the rapid development of the manufacture of composite materials with desired properties, the creation of composite materials with specified optimal physical and mechanical characteristics comes to the fore.Layered composite plates and shells are widely used as light elements in structures in the aviation, shipbuilding, and urban planning industries as load-bearing elements with improved physical, mechanical, and thermal characteristics. For example, developments are underway to create fire-, heat- and moisture-resistant materials, plates, and shallow shells for the safety of fire service personnel [1-6]. Here we propose a new approach to constructing a non-classical theory of temperature propagation in layered composite plates.

MAIN PART

Temperature field of a three-dimensional body under the influence of a system of volumetric sources of temperature and surface thermal effects is described in the Cartesian coordinate system $Ox_1x_2x_3$ by the following heat conduction equation [7-10]

$$\left(\lambda_{ij}\theta,_j\right),_i + f(x,y,z) = 0 \tag{1}$$

with linear defining relations between heat q_i and temperature gradient θ

$$q_i = \lambda_{ij}\theta,_j \tag{2}$$

when the following boundary conditions are met

$$\theta\big|_{\Sigma} = \theta_{\Sigma}, \qquad q_i n_i\big|_{\Sigma} = q_{\Sigma} \tag{3}$$

where $f(x,y,z)$ are the volumetric temperature sources, θ_{Σ} is the temperature set on the surface, q_{Σ} is the heat flux along the normal coordinate, n_i are the components of the outer normal, λ_{ij} are the thermal conductivity coefficients, $i,j,k,l = 1,2,3$.

Let the Cartesian coordinate system Ox_1x_2z be located in the lower plane of a layered composite plate with constant thickness *H*. The dependences of the elastic moduli of the layered plate along the z coordinate are given as

The International Scientific and Practical Conference RAKHMATULIN READINGS
AIP Conf. Proc. 3119, 020005-1–020005-7; https://doi.org/10.1063/5.0214779
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$$\lambda_{ij}(x_1,x_2,z)=\lambda^{(1)}{}_{ij}(x_1,x_2)\chi(z_1-z)+\lambda^{(N)}{}_{ij}(x_1,x_2)\chi(z-z_{N-1})+\sum_{r=2}^{N-1}\lambda^{(r)}{}_{ij}(x_1,x_2)\chi(z_r-z)\chi(z-z_{r-1}) \quad (4)$$

where z_r are the coordinates of the interface of composite plate layers. Here $h_r = z_{r+1} - z_r$, $r = 2,3,\cdots N-1$, $z_1 = 0$, $z_N = H$, and $\chi(z)$ is the Heaviside function.

The sought-for solution to problem (1)-(3), considering (4), is represented in the following form:

$$\theta(x_1,x_2,z)=\theta^{(1)}(x_1,x_2,z)\chi(z_1-z)+\theta^{(N)}(x_1,x_2,z)\chi(z-z_{N-1})+\sum_{r=2}^{N-1}\theta^{(r)}(x_1,x_2,z)\chi(z_r-z)\chi(z-z_{r-1}) \quad (5)$$

The following system of the heat equation is fulfilled for each "package":

$$\left(\lambda^{(r)}{}_{ij}\theta^{(r)},_j\right),_i + f = 0 \quad (6)$$

For each r-"package" we introduce "quick" variable ξ using the change of variables $z = H\xi$. The sought-for solution for each r layer is represented as a polynomial in ξ,

$$\theta^{(r)} = \sum_{n=0}^{\infty} Z_n{}^{(r)}(\xi)\varphi_n(x) \quad (7)$$

where

$$Z_n{}^{(r)}(\xi) = C_n{}^{(r)}ch(\tilde{\gamma}_n\xi) + D_n{}^{(r)}ch(\tilde{\gamma}_n\xi),$$

here $C_n{}^{(r)}$ and $D_n{}^{(r)}$ are unknown constants to be determined from the boundary conditions along the normal coordinate under the influence of the temperature field. At the interface $z = z_r$ the conditions of continuity for the temperature fields are satisfied for each *r*-"package":

$$\theta^{(r)} = \theta^{(r+1)},\ q_z^{(r)} = q_z^{(r+1)}, \quad r = 1,2,\cdots,N-1 \quad (8)$$

As an applied problem, consider the following case:

$$\begin{cases} \dfrac{\partial\theta^{(1)}}{\partial z} + \alpha\theta^{(1)} = 0, & for \quad \xi = 0, \\ q_z{}^{(1)} = q_z{}^{(2)},\ \theta^{(1)} = \theta^{(2)}, & for \quad \xi = \dfrac{1}{2+k}, \\ q_z{}^{(3)} = q_z{}^{(2)},\ \theta^{(3)} = \theta^{(2)}, & for \quad \xi = \dfrac{1+k}{2+k}, \\ \theta^{(3)} = \theta_{\Sigma}, & for \quad \xi = 1. \end{cases} \quad (9)$$

We assume that a laminated three-layer plate consists of a "package" of composite materials. In this case, each "package" under consideration is an isotropic medium with different thermal conductivity coefficients $\lambda^{(r)}{}_{ij} = \lambda^{(r)}\delta_{ij}$ and different thickness h_r, here $h_1 = h_3 = h$, $h_2 = kh$, and $h_1 + h_2 + h_3 = H$. In particular, if $k = 0$, we obtain a two-layer plate, and if $\lambda^{(2)} = 0$ and $k = 1$, we have a three-layer plate with a heat-insulating inner layer.

For this problem, we introduce into consideration "fast" coordinate ξ: $z = H\xi$, $h = \frac{H}{2+k}$.

On the remaining faces, convective heat transfer $\left.\dfrac{\partial\theta^{(1)}}{\partial z} + \alpha\theta^{(1)}\right|_{\Sigma} = 0$ can take place or the ambient temperature $\theta^{(r)} = 0$ is set. Let there be no volumetric heat sources in the layered composite $f(x,y,z) = 0$.

Let us consider some partial cases:

For $k = 0$, it follows from conditions (9) that instead of a three-layer composite, we have a two-layer composite.

$$\begin{cases} \dfrac{\partial \theta^{(1)}}{\partial z} + \alpha \theta^{(1)} = 0, & \xi = 0, \\ {q_z}^{(1)} = {q_z}^{(2)},\ \theta^{(1)} = \theta^{(2)}, & \xi = \dfrac{1}{2}, \\ \theta^{(2)} = \theta_{\Sigma}, & \xi = 1. \end{cases} \tag{10}$$

As $k \to \infty$, it follows from conditions (9) that the problem under consideration corresponds to the one-layer option.

$$\begin{cases} \dfrac{\partial \theta^{(1)}}{\partial z} + \alpha \theta^{(1)} = 0, & \xi = 0, \\ \theta^{(1)} = \theta_{\Sigma}, & \xi = 1. \end{cases} \tag{11}$$

Let θ_{Σ} be expressed as follows

$$\theta_{\Sigma} = \sum_{n=0}^{\infty} q_n \varphi_n(x), \tag{12}$$

where $q_n = 2\theta_{\Sigma} \int_0^1 \varphi_n(x) dx$ [11-12]. The form of the function depends on the boundary conditions. With boundary conditions $\theta^{(r)} = 0$ for $x = 0, l$, the basis functions take the following form $\varphi_n(x) = \sin \dfrac{\pi(2n+1)}{l} x$, and the boundary conditions for convective heat transfer for $x = 0, l$ are satisfied if the basis function has the form of a sine polynomial

$$\varphi_n(x) = \sin \frac{\pi n}{l} x - \frac{1}{3} \sin \frac{3\pi n}{l} x .$$

For the boundary conditions at , the basis functions will take the form: , the boundary conditions for convective heat transfer at are satisfied if the basis function has the form of a sine polynomial .

If we assume that there is a heat-insulating material, that does not pass the temperature flux, then the solution to the problem becomes critical, and a new notation is introduced:

$$\theta^{(r)} = \frac{\tilde{\theta}^{(r)}}{\lambda^{(r)}} \tag{13}$$

With (13), system (9) can be written as follows

$$\begin{cases} \lambda^{(1)} \dfrac{\partial \tilde{\theta}^{(1)}}{\partial z} + \alpha \tilde{\theta}^{(1)} = 0, \quad \xi = 0, \\ {q_z}^{(1)} = {q_z}^{(2)},\ \lambda^{(2)} \tilde{\theta}^{(1)} = \lambda^{(1)} \tilde{\theta}^{(2)},\ \xi = \dfrac{1}{2+k}, \\ {q_z}^{(3)} = {q_z}^{(2)},\ \lambda^{(2)} \tilde{\theta}^{(3)} = \lambda^{(3)} \tilde{\theta}^{(2)},\ \xi = \dfrac{1+k}{2+k}, \\ \tilde{\theta}^{(3)} = \lambda^{(3)} \theta_{\Sigma}, \quad \xi = 1. \end{cases} \tag{14}$$

The solution to system (9) for the three-layer case can be written as follows:

$$\begin{cases} \theta^{(1)} = \sum_{n=0}^{\infty} \frac{\lambda^{(2)}\lambda^{(3)} q_n \left(Bch\left(\frac{\tilde{\gamma}_n}{2+k}\right) - Ash\left(\frac{\tilde{\gamma}_n}{2+k}\right)\right)}{P\left(\lambda^{(1)} sh\left(\frac{\tilde{\gamma}_n}{2+k}\right) - \frac{\alpha}{\tilde{\gamma}_n} ch\left(\frac{\tilde{\gamma}_n}{2+k}\right)\right)} \left(ch(\tilde{\gamma}_n \xi) - \frac{\alpha}{\tilde{\gamma}_n} sh(\tilde{\gamma}_n \xi)\right)\varphi_n(x) \\ \theta^{(2)} = \sum_{n=0}^{\infty} \frac{\lambda^{(3)} q_n}{P}\left(Bsh(\tilde{\gamma}_n\xi) - Ach(\tilde{\gamma}_n\xi)\right)\varphi_n(x) \\ \theta^{(3)} = \sum_{n=0}^{\infty} \frac{q_n \left(\left(1 - \frac{\lambda^{(2)} sh(\tilde{\gamma}_n)}{P} G\right) ch(\tilde{\gamma}_n\xi) + \left(\frac{\lambda^{(2)} ch(\tilde{\gamma}_n)}{P} G - th\left(\frac{1+k}{2+k}\tilde{\gamma}_n\right)\right) sh(\tilde{\gamma}_n\xi)\right)}{\left(ch(\tilde{\gamma}_n) - sh(\tilde{\gamma}_n) th\left(\frac{1+k}{2+k}\tilde{\gamma}_n\right)\right)} \varphi_n(x) \end{cases} \tag{15}$$

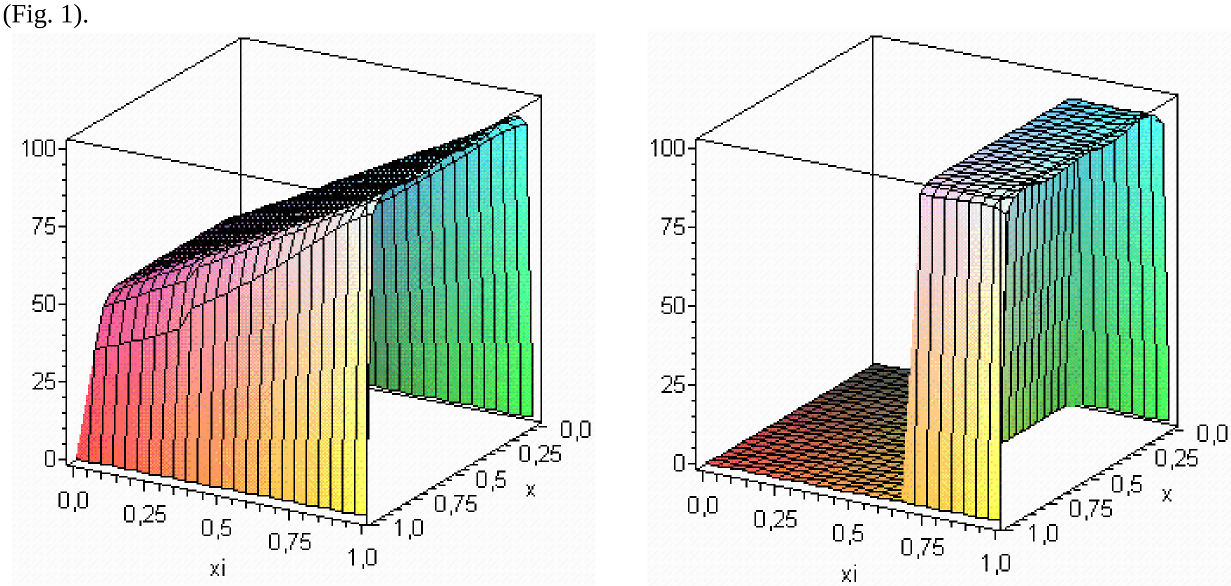
where $A = \left(\lambda^{(2)} - \lambda^{(1)}\right)\left(\lambda^{(1)} ch\left(\frac{2\tilde{\gamma}_n}{2+k}\right) - \frac{\alpha}{\tilde{\gamma}_n} sh\left(\frac{2\tilde{\gamma}_n}{2+k}\right)\right) + \lambda^{(1)}\left(\lambda^{(1)} + \lambda^{(2)}\right)$,

$B = \left(\lambda^{(2)} - \lambda^{(1)}\right)\left(\lambda^{(1)} sh\left(\frac{2\tilde{\gamma}_n}{2+k}\right) - \frac{\alpha}{\tilde{\gamma}_n} ch\left(\frac{2\tilde{\gamma}_n}{2+k}\right)\right) + \frac{\alpha}{\tilde{\gamma}_n}\left(\lambda^{(1)} + \lambda^{(2)}\right)$, $G = B - Ath\left(\frac{1+k}{2+k}\tilde{\gamma}_n\right)$,

$$P = \lambda^{(3)} ch\left(\frac{\tilde{\gamma}_n}{2+k}\right)\left(Bsh\left(\frac{1+k}{2+k}\tilde{\gamma}_n\right) - Ach\left(\frac{1+k}{2+k}\tilde{\gamma}_n\right)\right) + \lambda^{(2)} sh\left(\frac{\tilde{\gamma}_n}{2+k}\right)\left(Bch\left(\frac{1+k}{2+k}\tilde{\gamma}_n\right) - Ash\left(\frac{1+k}{2+k}\tilde{\gamma}_n\right)\right)$$

DISCUSSION OF RESULTS

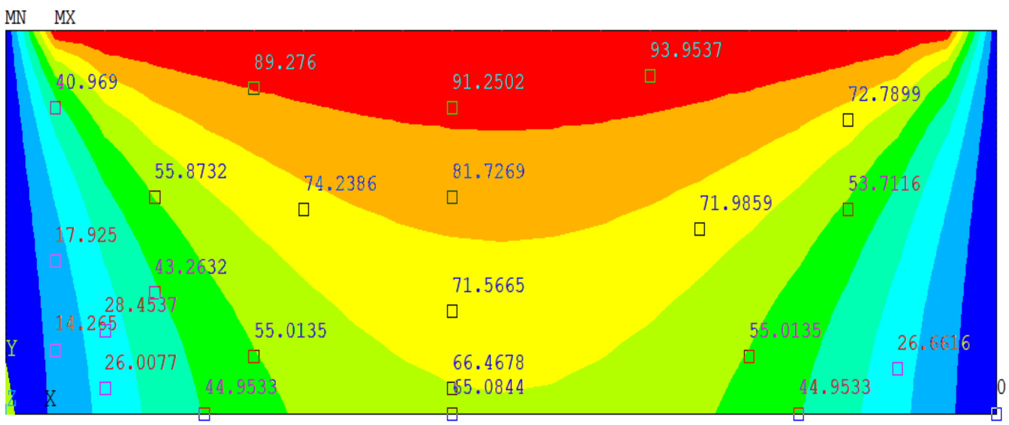
When the coefficient of convective heat transfer $\alpha = 1$ is given on the boundary $\theta_\Sigma = 100^\circ$, the problem is considered for a three-layered plate with different values of the thermal conductivity of the middle layer $\lambda^{(2)} = 1;0$. (Fig. 1).



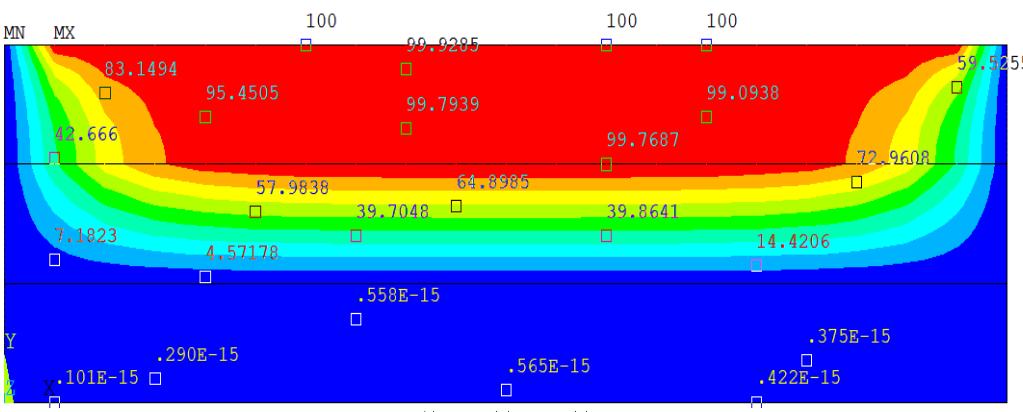
a) $\lambda^{(1)} = 1, \lambda^{(2)} = 1, \lambda^{(3)} = 1$ b) $\lambda^{(1)} = 1, \lambda^{(2)} = 0, \lambda^{(3)} = 1$

FIGURE 1. Graph of temperature distribution in a three-layer plate for $\alpha = 1$; $k = 1$ obtained by the appropriate program Maple 12: a) $\lambda^{(2)} = 1$, b) $\lambda^{(2)} = 0$.

On fig. 1 shows a graph of the distribution of the temperature field obtained by an analytical solution using the Maple 12 application program. The temperature was chosen as a function expanded by a sine polynomial. To compare the above solutions, using the Ansys Mechanical APDL 2022 R1 application program based on the finite element method, temperature field distribution graphs were obtained (Fig. 2). The nature of the solutions obtained for a constant temperature with respect to the corresponding basis functions becomes trivial at the edges; this feature is taken into account in *Ansys*. The results obtained are in satisfactory agreement. For example, in the lower plane of a three-layered plate for the coefficient of convective heat transfer $\alpha = 1$, the temperature values corresponding to the thermal conductivity coefficients $\lambda^{(2)} = 1$, $\lambda^{(2)} = 0{,}2$, $\lambda^{(2)} = 0$ are approximately equal to 50;0. The absence of temperature in the lower boundary of the lower plane of the three-layered plate is due to the presence of a heat-insulating middle layer.



a) $\lambda^{(1)} = 1, \lambda^{(2)} = 1, \lambda^{(3)} = 1$



b) $\lambda^{(1)} = 1, \lambda^{(2)} = 0, \lambda^{(3)} = 1$

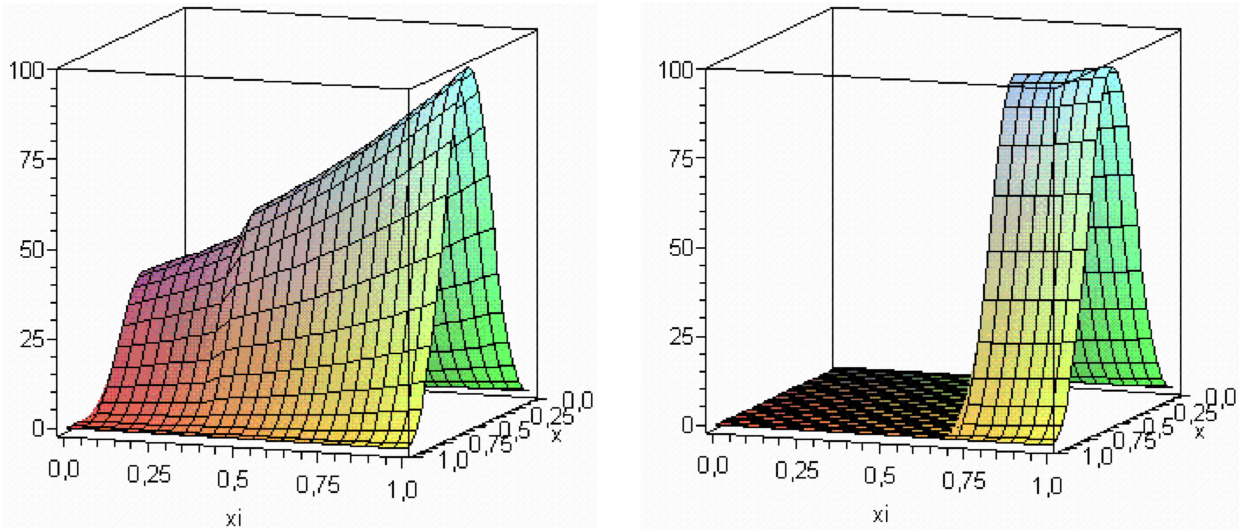
FIGURE 2. Graph of temperature distribution in a three-layer plate for $\alpha = 1$; $k = 1$ obtained by the appropriate program Ansys Mechanical APDL 2022 R1: a) $\lambda^{(2)} = 1$, b) $\lambda^{(2)} = 0$.

As follows from the results obtained, with a decrease in the thermal conductivity $\lambda^{(2)} \to 0$ of the middle layer, an abrupt decrease in temperature fields is observed, to its trivial value in the 1st and 2nd layers of the layered plate. This shows the theoretical possibility of creating a heat-insulating material.

The possibility of heat transfer at the edges of a three-layered plate leads to a redistribution in the direction of smoothing the resulting diagrams. The exception is the presence of a heat-insulating layer, where the temperature in the third layer does not spread to other layers.

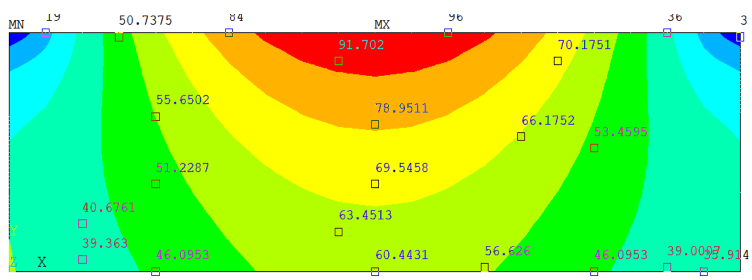
As $k \to 0$, the three-layered plate turns into a two-layered plate. In this case, for two outer layers, the thickness is

$h_1 = h_3$, and $h_2 \to 0$. This problem can be considered assuming that there is a very thin foil-like plate between the layers, which can be thermally insulating.

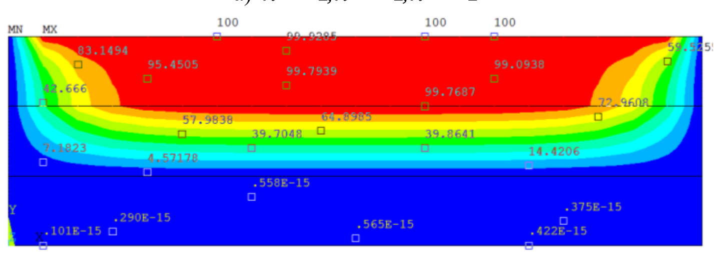


a) $\lambda^{(1)} = 1, \lambda^{(2)} = 1, \lambda^{(3)} = 1$ б) $\lambda^{(1)} = 1, \lambda^{(2)} = 0, \lambda^{(3)} = 1$

FIGURE 3. Temperature distribution under the influence of a parabolic load at possible heat transfer at the edges of a three-layered plate $\alpha = 1$; $k = 1$ obtained by the appropriate program Maple 12: a) $\lambda^{(2)} = 1$, b) $\lambda^{(2)} = 0$.



a) $\lambda^{(1)} = 1, \lambda^{(2)} = 1, \lambda^{(3)} = 1$



б) $\lambda^{(1)} = 1, \lambda^{(2)} = 0, \lambda^{(3)} = 1$

FIGURE 4. Temperature distribution under the influence of a parabolic load at possible heat transfer at the edges of a three-layered plate $\alpha = 1$; $k = 1$ obtained by the appropriate program *Ansys Mechanical APDL 2022 R1*: a) $\lambda^{(2)} = 1$, b) $\lambda^{(2)} = 0$.

On fig. 3 and fig. 4 shows the graphs of the distribution of the temperature field obtained by the analytical solution using the Maple 12 application program, as well as the Ansys Mechanical APDL 2022 R1 application program, based on the finite element method in the case of a parabolic load acting on a layered plate. The results obtained are in satisfactory agreement with the constant load shown above.

From the obtained results, it follows that the presence of heat-insulating foil in the middle of the walls can lead to a positive effect concerning comfortable conditions in areas with particularly hot weather or in the Far North, resulting in cost-effective urban development.

CONCLUSIONS

1. An effective approach to solving problems of heat conduction in multilayer plates was proposed;
2. The presence of a heat-insulating layer in multilayer structures can significantly change the pattern of temperature field distribution;
3. Based on the results obtained, a theoretical basis for designing multilayer structures with desired properties was developed.
4. Using the Maple 12 and Ansys Mechanical APDL 2022 R1 application programs, graphs of the distribution of the temperature field were obtained.

REFERENCES

1. Olenenko M., Velichko I. Analytical solution of a problem of the stationary distribution of heat in anisotropic multilayer plates. //New materials and technologies in metallurgy and mechanical engineering (ISSN 1607-6885) №1, 2013, 88-92 [in Ukrainian].
2. V.Havrysh, L.Koliasa, S.Vozna. Temperature field in a layered plate with local heating. International scientific journal "Mathematical modeling" (WEB ISSN 2603-2929; PRINT ISSN 2535-0986) ISSUE 3, pp. 90-94 (2021).
3. Yu. B. Protsyuk, Static thermoelasticity problems for layered thermosensitive plates with cubic dependence of the coefficients of heat conductivity on temperature. Journal of Mathematical Sciences, Vol. 181, No. 4, March, pp.481-496 (2012).
4. Vlasov V.V. The method of initial functions in problems of the theory of elasticity and structural mechanics / Vladimir Vasilievich Vlasov. - M: Stroyizdat, 1975. - 223 p.
5. V.Havrysh. Mathematical spatial models of determination of temperature field from locally concentrated thermal heating. Ukrainian journal of information technology, Vol. 4, No.1 pp. 21-28 (2022).
6. Sergey B. et al. A technique for evaluation of tank boiler stability by the variational method //AIP Conference Proceedings. – AIP Publishing, 2022. – T. 2637. – №. 1.
7. Novatsky V. Dynamic problems of thermoelasticity. -M.: Mir, 1970. - 256 p.
8. Parton V.Z. Methods of mathematical theory of elasticity / V. Parton, P. Perlin - M.: Nauka, 1981- 688 p.
9. Kovalenko A.D. Fundamentals of thermoelasticity / Anatoly Dmitrievich Kovalenko. - K.: Naukova Dumka, 1970. - 309 p.
10. V. Kech, P.Teodorescu. Introduction to the theory of generalized functions with applications in engineering. - M: Mir, 1978, 520 p.
11. Vorobyov N.N. Theory of series / N.N. Vorobyov. - M: Nauka, 1979. – 198 p.
12. Sneddon I. Fourier transform / I. Sneddon - M.: Publishing house of foreign literature, 1955. – 688 p.