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$A(z)$-GARMONIK FUNKSIYALAR

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REZYUME

Ushbu maqolada $A(z)$-garmonik funksiyalarga ta'rif berilib ularni $A(z)$-analitik funksiyalar bilan bog'lanishi keltirilgan.

Kalit so'zlar: $A(z)$-garmonik funksiya, umumlashgan $A(z)$-garmonik funksiya, $A(z)$-analitik funksiya.

Kompleks tekislikdagi D sohada analitik bo'lgan $A(z)$ funksiya berilgan bo'lib, barcha $z \in D$ uchun $|A(z)| \leq C < 1$ tengsizlik o'rinli bo'lsin. $u : D \to R$, $u \in C^2(D)$ funksiya D sohada $A(z)$-garmonik funksiya deyiladi, agar barcha $z \in D$ uchun

$$\frac{\partial}{\partial z}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z}\right]\right] + \frac{\partial}{\partial \bar{z}}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial u}{\partial z} - 2\overline{A}\frac{\partial u}{\partial \bar{z}}\right]\right] = 0$$

tenglik o'rinli bo'lsa.

$\Delta_A : C^2(D) \to C(D)$ orqali quyidagi differensial operatorni belgilaymiz:

$$\Delta_A u = \frac{\partial}{\partial z}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial}{\partial \bar{z}} - 2\overline{A}\frac{\partial}{\partial z}\right]\right] +$$

$$+\frac{\partial}{\partial \bar{z}}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial}{\partial z} - 2A\frac{\partial}{\partial \bar{z}}\right]\right]. \tag{1}$$

$$\Delta_A = \frac{1}{2}\mathrm{Re}\frac{\partial}{\partial z}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial}{\partial \bar{z}} - 2\overline{A}\frac{\partial}{\partial z}\right]\right]$$

bo'lgani uchun bu operator haqiqiy operator bo'ladi. $A(z) = 0$ bo'lganda Δ_A klassik Laplas operatori bilan ustma ust tushadi ([2]-[5] ga qarang).

Masalan: $u(z) = 2(z+\bar{z}) + z^2 + \bar{z}^2$ funksiyani ko'rsak $A(z) = \bar{z}$ va $D = \{|z| < \frac{1}{2}\}$ uchun $A(z)$-garmonik funksiya bo'ladi.

Teorema 1. *Agar f funksiya D sohada $A(z)$ analitik funksiya ([1], [3], [4] ga qarang) bo'lsa u holda uning haqiqiy va mavhum qismlari D sohada $A(z)$-garmonik funksiya bo'ladi.*

Isboti. $f \in O_A(D)$ bo'lgani uchun quyidagi tengliklar o'rinli:

$$\begin{cases} \frac{\partial f}{\partial \bar{z}} = A\frac{\partial f}{\partial z} \\ \frac{\partial \bar{f}}{\partial z} = \overline{A}\frac{\partial \bar{f}}{\partial \bar{z}} \end{cases} \Leftrightarrow \begin{cases} \frac{\partial u}{\partial \bar{z}} + i\frac{\partial v}{\partial \bar{z}} = A(\frac{\partial u}{\partial z} + i\frac{\partial v}{\partial z}) \\ \frac{\partial u}{\partial z} - i\frac{\partial v}{\partial z} = \overline{A}(\frac{\partial u}{\partial \bar{z}} - i\frac{\partial v}{\partial \bar{z}}) \end{cases}.$$

Yuqoridagi tenglamalar sitemasini $\frac{\partial v}{\partial z}$ va $\frac{\partial v}{\partial \bar{z}}$ ga nisbatan yechib

$$\frac{\partial v}{\partial z} = -\frac{i}{1-|A|^2}\left[(1+|A|^2)\frac{\partial u}{\partial z} - 2\overline{A}\frac{\partial u}{\partial \bar{z}}\right],$$

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$$\frac{\partial v}{\partial \bar{z}} = \frac{i}{1-|A|^2}\left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z}\right]$$

tengliklarga ega bo'lamiz. $v \in C^2$ bo'lgani uchun aralash hosilalari teng va

$$0 = \frac{\partial^2 v}{\partial z \partial \bar{z}} - \frac{\partial^2 v}{\partial \bar{z} \partial z} = i\frac{\partial}{\partial z}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z}\right]\right] +$$

$$+i\frac{\partial}{\partial \bar{z}}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial u}{\partial z} - 2\overline{A}\frac{\partial u}{\partial \bar{z}}\right]\right] = -i\Delta_A u$$

tenglik o'rinli bo'lib, bundan $\Delta_A u = 0$ ya'ni $A(z)$ analitik funksiyani haqiqiy qismi $A(z)$ garmonik ekanligi kelib chiqadi. $-if = v - iu \in O_A(D)$ bo'lgani uchun xuddi yuqoridagi kabi $\Delta_A v = 0$ bo'lishini ko'rsatish mumkin.

Teorema 2. *Agar u funksiya biror $r > 0$ uchun*

$$L = L(a,r) = \left\{ z \in \mathbb{C} : \left| z - a + \int_{\gamma(a,z)} \overline{A}(w)dw \right| < r \right\} \Subset D$$

lemniskatada $A(z)$-garmonik funksiya bo'lsa u holda shunday $f \in O_A(L)$ funksiya topiladiki $u(z) = Ref(z)$ o'rinli bo'ladi.

Isboti. Ushbu d^c_A differensial operatorni ko'raylik:

$$d^c_A u = -\frac{i}{1-|A|^2}\left[((1+|A|^2)\frac{\partial u}{\partial z} - 2\overline{A}\frac{\partial u}{\partial \bar{z}})dz - ((1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z})d\bar{z}\right] \quad (2)$$

U holda

$$dd^c_A u = -i\frac{\partial}{\partial z}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z}\right]\right] +$$

$$+i\frac{\partial}{\partial \bar{z}}\left[\frac{1}{1-|A|^2}\left[(1+|A|^2)\frac{\partial u}{\partial z} - 2\overline{A}\frac{\partial u}{\partial \bar{z}}\right]\right] dz \wedge d\bar{z} = -i\Delta_A u dz \wedge d\bar{z} = 2\Delta_A u dV.$$

$$\Delta_A u dV = -\frac{i}{2}\Delta_A u dz \wedge d\bar{z} = \frac{1}{2} dd^c_A u. \quad (3)$$

u funksiya $A(z)$-garmonik bo'lgani uchun (3) tenglikka ko'ra $dd^c_A u$ differensial forma yopiq bo'ladi. $z \in L$ uchun

$$\int_a^z d^c_A u$$

integralni qiymati integrallash yo'liga bog'liq emas. Chunki ikkita gomotop $\gamma_1(a,z)$ va $\gamma_2(z,a)$ yo'llarni ko'rsak va $\partial G = \gamma = \gamma_1(a,z) \cup \gamma_2(a,z)$ belgilasak, Stoks formulasiga ko'ra

$$\int_{\gamma_1(a,z)} d^c_A u - \int_{\gamma_2(z,a)} d^c_A u = \oint_\gamma d^c_A u = \iint_G dd^c_A u = -2\iint_G \Delta_A u dV = 0$$

bo'lib integrallarni qiymati teng bo'ladi.

Endi

$$v(z) = \int_a^z d^c_A u$$

funksiyani qaraymiz. Bu funksiya argumentiga $z = x + iy$ almashtirish kiritsak,

$$v(z) = \int_a^z \left(\frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right] + \frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial z} - 2\bar{A}\frac{\partial u}{\partial \bar{z}} \right] \right) dx +$$

$$+i \int_a^z \left(\frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right] - \frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial z} - 2\bar{A}\frac{\partial u}{\partial \bar{z}} \right] \right) dy =$$

$$= \int_a^z \left(\frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right] + \overline{\frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right]} \right) dx +$$

$$+i \int_a^z \left(\frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right] - \overline{\frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right]} \right) dy =$$

$$= 2\int_a^z \mathrm{Re} \left(\frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right] \right) dx + 2\int_a^z \mathrm{Im} \left(\frac{1}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right] \right) dy$$

ko'rinishiga kelib, funksiyaning qiymatlari haqiqiy ekanligi kelib chiqdi. Shuningdek

$$\frac{\partial v}{\partial z} = -\frac{i}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial z} - 2\bar{A}\frac{\partial u}{\partial \bar{z}} \right]$$

$$\frac{\partial v}{\partial \bar{z}} = \frac{i}{1-|A|^2} \left[(1+|A|^2)\frac{\partial u}{\partial \bar{z}} - 2A\frac{\partial u}{\partial z} \right]$$

tengliklar o'rinli bo'lib, $f(z) = u(z) + iv(z)$ funksiyani qursak barcha $z \in D$ uchun

$$D_A f = \frac{\partial f}{\partial \bar{z}} - A\frac{\partial f}{\partial z} = \frac{\partial u}{\partial \bar{z}} + i\frac{\partial v}{\partial \bar{z}} - A\left(\frac{\partial u}{\partial z} + i\frac{\partial v}{\partial z} \right) = 0$$

tenglikni qanoatlantiradi. Demak $f \in O_A(L)$.

Natija. *D sohada $A(z)$ – cheksiz marta differensiallanuvchi bo'ladi.*

Lemma. *$u, v \in C^1(D)$ funksiyalar uchun quyidagi tenglik o'rinli:*

$$d_A^c u \wedge dv = d_A^c v \wedge du.$$

Ta'rif. *Agar barcha $\varphi \in F(D)$ finit test funksiya uchun*

$$\Delta_A u(\varphi) = u(\Delta_A \varphi) = \int_D u \Delta_A \varphi dV = 0 \tag{4}$$

tenglik bajarilsa, u holda $u \in L^1_{loc}(D)$ funksiya D sohada umumlashgan ma'noda $A(z)$-garmonik funksiya deyiladi.

Agar u funksiya D sohada $A(z)$-garmonik funksiya bo'lsa u holda (4) tenglik, lemma hamda Stoks formulasiga ko'ra

$$\Delta_A u(\varphi) = \int_D \Delta_A u \varphi dV = -\frac{1}{2}\int_D dd_A^c u \varphi = -\frac{1}{2}\int_D d(d_A^c u \varphi) + \frac{1}{2}\int_D d_A^c u \wedge d\varphi =$$

$$= \frac{1}{2}\int_D d^c_A\varphi \wedge du = -\frac{1}{2}\int_D du \wedge d^c_A\varphi = -\frac{1}{2}\int_D d(u \wedge d^c_A\varphi) + \frac{1}{2}\int_D udd^c_A\varphi =$$

$$= \frac{1}{2}\int_D udd^c_A\varphi = \frac{1}{2}\int_D u\Delta_A\varphi dV$$

tengliklar o'rinli bo'lib, bu tenglikdan $A(z)$-garmonik funksiya umumlashgan ma'noda ham $A(z)$-garmonik funksiya bo'lishi kelib chiqadi.

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РЕЗЮМЕ

В данной статье вводится понятие $A(z)$-гармонической функции и получены отношения между $A(z)$-аналитическими и $A(z)$-гармоническими функциями.

Ключевые слова: $A(z)$-гармоническая функция, обобщенная $A(z)$-гармоническая функция, $A(z)$-аналитическая функция.

RESUME

In this work we define $A(z)$-harmonic function and founded relation between $A(z)$-analytic function with $A(z)$-harmonic function.

Key words: $A(z)$-harmonic function, generalized $A(z)$-harmonic function, $A(z)$-analytic function.