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COMPUTER SIMULATION METHODS FOR APPLIED MATHEMATICS PROBLEMS

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This monograph presents a comprehensive study of computer simulation methods for a wide class of problems in applied mathematics, based on modern numerical approaches and computational algorithms. The work focuses on the development and application of mathematical models and numerical techniques for the analysis of complex physical and engineering processes, with particular emphasis on the finite element method (FEM) as a universal tool for solving boundary-value and initial-boundary-value problems. Special attention is given to the theoretical foundations of numerical discretization, including variational formulations, finite element approximation, and the treatment of multiply connected and complex computational domains. Modern mesh generation techniques and algorithms for constructing finite element meshes for complex and composite geometries are discussed in detail, providing a solid mathematical and algorithmic basis for computer simulations. A significant part of the monograph is devoted to the numerical solution of nonstationary problems described by partial differential equations, including heat conduction and diffusion-type models in heterogeneous and inhomogeneous media. Two-dimensional and axisymmetric formulations are considered, as well as problems involving complex boundary conditions, contact interactions, and coupled physical effects. Computational experiments illustrate the influence of model parameters, material heterogeneity, and boundary conditions on the behavior of numerical solutions.

The book further addresses coupled and multiphysics problems of applied mathematics, including quasi-static and transient thermoelasticity, elasticity, and deformation mechanics. Finite element models and numerical algorithms are developed for the analysis of stress–strain states, axisymmetric problems, and domains containing geometric and physical singularities. These examples demonstrate the effectiveness of computer simulation in studying complex mathematical models arising in continuum mechanics. In addition, selected chapters are devoted to the numerical modeling of flow and transport processes in porous media, including the computation of scalar potential fields, which illustrates the versatility and broad applicability of the proposed computational framework to different classes of applied mathematics problems. The final part of the monograph focuses on numerical methods for solving nonlinear problems, including elastic and elastoplastic models. Stress concentration phenomena, the influence of geometric irregularities, and the convergence and stability of numerical algorithms are analyzed through representative computational examples. The results confirm the efficiency, accuracy, and robustness of the developed simulation methods.

The monograph is intended for researchers, graduate students, and engineers specializing in applied mathematics, numerical analysis, computational mechanics, and scientific computing.

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The monograph is recommended for publication by the decision of the Academic Council of the National University of Uzbekistan.

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INTRODUCTION

At present, structural elements made of modern structural materials have become widespread in various fields of technology and construction. They include homogeneous and composite structures of complex configurations and have wide mechanical, thermal, and physical properties. The rapidly expanding use of composites in load-bearing structures for various purposes requires developing calculation models and methods that consider the structural features of these materials. These features include thermal properties and anisotropy, which must be accounted for when calculating structures for thermal properties and strength. Effective implementation of the advantages of composite materials in structures requires solving a set of problems related to determining the rational structure of the material corresponding to the field of external loads and temperature influences. The solution to engineering problems, one way or another, is associated with determining the strength and reliability of load-bearing structural elements, which, in turn, is reduced to solving problems of non-stationary thermal conductivity and physically nonlinear deformation of structural elements made of composite materials.

The active practical use of various structural elements from composite materials has contributed to developing the research in thermal conductivity and anisotropic plasticity theory. Numerical modeling of the nonlinear behavior of structural materials based on a computational experiment, by selecting temperature and mechanical parameters and the volumetric content of fiber and binder matrix in the composite, allows the creation of composite materials with predetermined properties.

For this purpose, in the monograph, based on the mathematical modeling apparatus, numerical modeling of the unsteady process of thermal conductivity and physically nonlinear deformation of structural materials was developed. The theory of small elastoplastic deformations for transversely isotropic media is used to describe these processes in fibrous composite materials.

Numerical modeling technology for studying phenomena or processes consists of constructing an adequate mathematical model, developing methods and algorithms for solving applied problems and conducting computational experiments based on existing or specially created software. For a computational experiment, a numerical model is used, which implements an abstract model of some system and is designed to obtain new knowledge about the modeled object. This also applies to the problems of studying heat distribution and strength of structural elements. To reduce the design time and minimize material costs, it is important to use a numerical modeling methodology, which allows, through computational experiments, to analyze and interpret the modeling results. This considers the configuration, thermal characteristics, and heterogeneity of composite materials.

To provide the user with a convenient interface for describing a real process, conducting computational experiments, and visualizing the results of calculations, it is necessary to create new and modify existing computational algorithms, based on which the software complex is built. Conducting experiments on a computer makes it possible to design composite materials with new structures and to study the influence of temperature effects, volumetric ratio, and mechanical parameters of the fiber and matrix on the strength of the structure. Thus, it can be argued that the development of effective algorithms for solving applied problems and the use of numerical modeling technology based on a specialized software package in the study of temperature effects and physically nonlinear deformation processes is relevant since it becomes possible to design new structural materials and evaluate the strength characteristics and reliability of the elements of developed designs.

Research in developing composite materials is an intensively developing field of science, where theoretical and practical research yields significant results. In this regard, numerical modeling of solving problems of thermal conductivity, thermoelasticity, and physically nonlinear deformation of transversely isotropic bodies for the reliable assessment of the strength resources of structural elements of complex configuration with stress concentrators based on modern computer technologies is relevant for the design and construction of modern structural materials.

The monograph is organized as a coherent and logically structured presentation of the theoretical and applied aspects of computer modeling based on the finite element method. Chapter 1 is devoted to the theoretical foundations of the finite element method. It addresses the mathematical formulation of problems in multiply connected three-dimensional domains, the selection of approximation schemes and basis functions, and modern techniques for finite element mesh generation. Algorithms for constructing meshes for complex and composite geometries are presented, which are essential for the numerical analysis of real engineering structures. Chapters 2 and 3 focus on finite element models of nonstationary heat transfer processes in composite and heterogeneous media. Special attention is paid to the treatment of material inhomogeneity, anisotropy, and complex boundary conditions. Numerical examples of heat transfer problems are provided, and the versatility of the proposed approaches is demonstrated through their application to groundwater flow modeling. Chapter 4 is dedicated to the computer simulation of thermoelasticity and classical elasticity problems. The stress–strain states of bodies subjected to combined mechanical and thermal loading are analyzed. Particular emphasis is placed on the modeling of perforated and nonuniform bodies, which makes it possible to investigate the influence of geometry and structural features on deformation behavior and stress distribution. Chapter 5 examines elastic and elastoplastic deformation in fiber-reinforced composites. The mechanisms of stress concentration are studied, along with the effects of reinforcement architecture and geometric factors on the mechanical behavior and strength characteristics of composite materials. Numerical examples illustrate the capabilities of the finite element method for analyzing complex stress–strain states in composite structures. In the conclusion, the main results of the study are summarized, the principal findings are formulated, and promising directions for further development of numerical modeling methods in the fields of deformable solid mechanics and heat transfer are outlined.