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Exact solutions and stability analysis
in metric-graphs–based approach**

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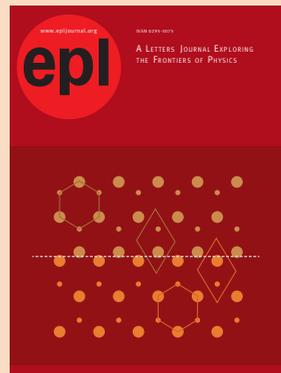
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Standing Manakov solitons in networks: Exact solutions and stability analysis in metric-graphs–based approach

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Abstract – We consider the (stationary) time-independent Manakov system on metric graphs. In particular, for the vertex boundary conditions following the conservation laws, we obtain exact soliton solutions of the problem. The ground-state solution, *i.e.*, the solution corresponding to a minimum of energy, is studied. The stability of the ground-state solution is analyzed using variational approaches.

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Introduction. – Optical solitons are fundamental carriers of physical characteristics that arise in various branches of modern optics and optoelectronics [1,2]. Among these, Manakov vector solitons hold a significant role in practical applications within nonlinear optics, condensed matter physics, and information technology. The dynamics of such solitons are governed by the Manakov system, an integrable system of coupled nonlinear Schrödinger equations (NLSEs), which admits one- and two-component vector soliton solutions. Various aspects of the Manakov system—including its integrability, soliton solutions, and practical implementations—have been extensively studied over the past few decades [3–17]. Its principal application lies in nonlinear optics, particularly in modeling optical vector solitons propagating through Kerr media [1,2]. Additionally, Manakov-type vector solitons have been observed in optical fibers [1,2,18], Bose-Einstein condensates [19], and other physical systems (see, *e.g.*, [1,2] for reviews). An intriguing application of Manakov solitons arises in photorefractive polymers and crystals [20–24]. Also, Manakov vector solitons garnered attention in the context of optical logic gates for computing purposes [10,11], and recent modifications of such logic gate concepts have been proposed in [25–27]. In [7], the experimental realization of Manakov solitons in crystals was demonstrated. The influence of small perturbations

on soliton collisions, relevant for ultrafast switching devices, was explored in [28]. The suppression of soliton interference in randomly birefringent optical fibers, due to vector soliton interactions, was studied in [29]. The rogue wave solutions of the variable-coefficient Manakov system with external potentials have been studied in [9]. The Manakov model has also been employed to describe beam propagation in Kerr media and AlGaAs waveguides [7], and to model matter waves in quantum systems. Properties of the energy-sharing Manakov solitons for the design of novel logic gates were analyzed in [10]. While an extensive body of literature exists on the traveling soliton solutions of the Manakov system, stationary (standing) Manakov solitons have received relatively little attention. Nevertheless, these stationary solutions are of considerable interest from both theoretical and applied perspectives, particularly in nonlinear optics and optoelectronics. Such solitons may be realized in finite photorefractive media—*e.g.*, crystals or polymers—with reflective boundaries such as mirrors. In this paper, we address the problem of standing Manakov solitons in networks by modeling them in terms of the time-independent Manakov system on metric graphs. These latter are composed of one-dimensional wires connected at vertices according to specific rules known as the graph topology [30,31]. Notably, evolution equations on graphs and the study of soliton dynamics in networks have attracted growing interest in recent years [32–44]. The paper is organized as follows:

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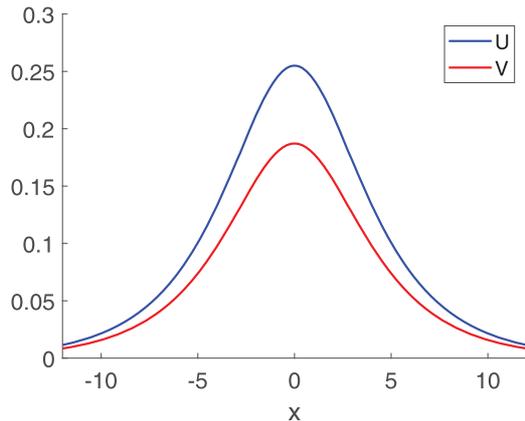


Fig. 1: Profile of Manakov solitons on the line for the parameters in eq. (5): $a = 0$, $\mu_1 = 0.065$, $\mu_2 = 0.035$.

the next section provides a brief overview of the Manakov system on the real line. The third section introduces the stationary Manakov system on graphs and presents explicit soliton solutions. The fourth section is devoted to the stability analysis of these stationary solutions. In the fifth section, the problem is extended to a tree graph. Possible experimental realization of the problem is discussed in the sixth section. The seventh section concludes the paper with a summary and outlook on future directions.

Manakov system on a line. – Here, following [23], we briefly recall the stationary Manakov system on a real line. It can be obtained from the time-dependent coupled nonlinear Schrödinger equations which are given by

$$\begin{aligned} i\frac{\partial u}{\partial t} + \frac{1}{2}\frac{\partial^2 u}{\partial x^2} + (|u|^2 + |v|^2)u &= 0, \\ i\frac{\partial v}{\partial t} + \frac{1}{2}\frac{\partial^2 v}{\partial x^2} + (|u|^2 + |v|^2)v &= 0. \end{aligned} \quad (1)$$

The norm and the energy for this system can be written as

$$\begin{aligned} M &= \int_{-\infty}^{+\infty} (|u|^2 + |v|^2) dx, \\ E &= \int_{-\infty}^{+\infty} \left(\frac{1}{2} \left| \frac{\partial u}{\partial x} \right|^2 + \frac{1}{2} \left| \frac{\partial v}{\partial x} \right|^2 - \frac{1}{2}|u|^4 - \frac{1}{2}|v|^4 - |u|^2|v|^2 \right) dx. \end{aligned} \quad (2)$$

Equation (1) approves factorization time- and space-variables by substitution [23]

$$\psi(x, t) = \exp\left(\frac{i}{2}\mu t\right) \Psi(x), \quad (3)$$

where $\psi(x, t) = \begin{pmatrix} u(x, t) \\ v(x, t) \end{pmatrix}$ and $\Psi(x) = \begin{pmatrix} U(x) \\ V(x) \end{pmatrix}$ and μ is a constant. This leads to the following stationary version of

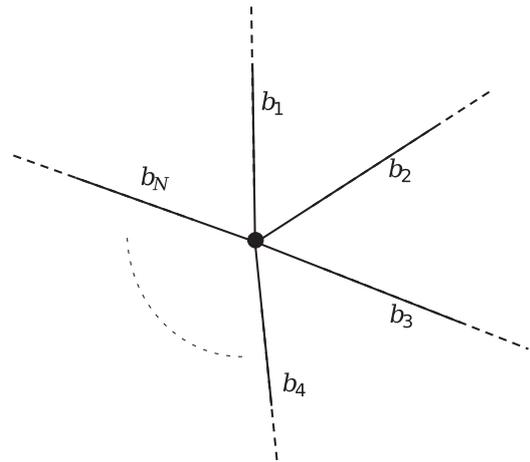


Fig. 2: Star graph with N bonds.

the Manakov system:

$$\begin{aligned} \frac{d^2 U}{dx^2} + 2(|U|^2 + |V|^2)U &= \mu U, \\ \frac{d^2 V}{dx^2} + 2(|U|^2 + |V|^2)V &= \mu V, \end{aligned} \quad (4)$$

with the asymptotic boundary conditions given by

$$U(x)|_{x \rightarrow \pm\infty} = 0, \quad V(x)|_{x \rightarrow \pm\infty} = 0.$$

Equation (4) approves the so-called “bright-bright” (standing) soliton solution given by [23]

$$\Psi(x) = \begin{pmatrix} \sqrt{\mu_1} \\ \sqrt{\mu_2} \end{pmatrix} \operatorname{sech}(\sqrt{\mu}(x - a)), \quad (5)$$

where $\mu = \mu_1 + \mu_2$ and a is the center of the mass. Equation (5) presents standing Manakov soliton on a line that can be generated, *e.g.*, using finite-size Kerr resonators [45] or photorefractive polymers [20–24]. In fig. 1, plots of the profiles of the solution given by eq. (5) are presented for the value of the parameter $a = 0$. The profile is symmetric with respect to the origin of coordinates. For nonzero values of a the symmetry is broken.

Stationary Manakov system on a star graph. – In this section, we consider the stationary Manakov system on the star graph with N bonds (see fig. 2), and x_j is assigned for bond b_j , where $j = 1, 2, \dots, N$. The coordinates are fixed as $x_j \in [0, +\infty)$. The wave function $\Psi_j(x_j) = \begin{pmatrix} U_j(x_j) \\ V_j(x_j) \end{pmatrix}$ is defined on each bond of the graph. In the following, for convenience we will use the shorthand notation x instead of x_j .

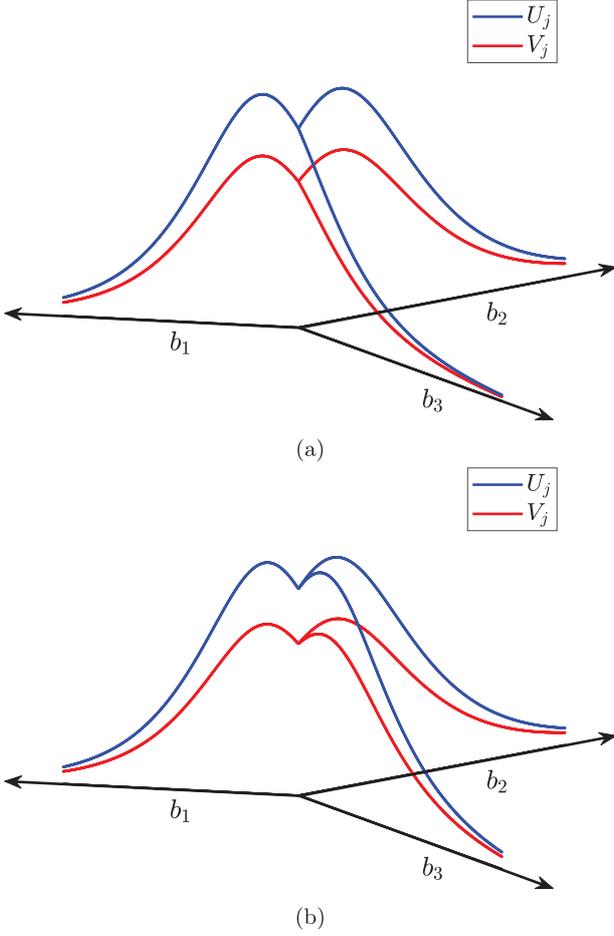


Fig. 3: Profile of standing Manakov solitons on a three-bond star graph plotted using the solutions (5) for $\lambda > 0$. The sign of $\{a_1, a_2, a_3\}$ is chosen as follows: $\{+, +, -\}$ for panel (a), and $\{+, +, +\}$ for panel (b). For both panels, the values of the parameters β_j and μ are chosen as $\beta_j = 1$, $\mu = 0.1$. The parameter a is calculated using eq. (13).

The stationary Manakov system on each bond of the star graph can be written as

$$\begin{aligned} \frac{d^2 U_j}{dx^2} + \beta_j (|U_j|^2 + |V_j|^2) U_j &= \mu U_j, \\ \frac{d^2 V_j}{dx^2} + \beta_j (|U_j|^2 + |V_j|^2) V_j &= \mu V_j, \end{aligned} \quad (6)$$

where β_j is the bond-dependent nonlinearity coefficient. Vertex boundary conditions can be written as wave function continuity

$$\alpha_1 \Psi_1|_{x=0} = \alpha_2 \Psi_2|_{x=0} = \dots = \alpha_N \Psi_N|_{x=0} \quad (7)$$

and current conservation law

$$\begin{aligned} \frac{1}{\alpha_1} \frac{d\Psi_1}{dx} \Big|_{x=0} + \frac{1}{\alpha_2} \frac{d\Psi_2}{dx} \Big|_{x=0} + \dots \\ + \frac{1}{\alpha_N} \frac{d\Psi_N}{dx} \Big|_{x=0} = \lambda \alpha_1 \Psi_1|_{x=0}, \end{aligned} \quad (8)$$

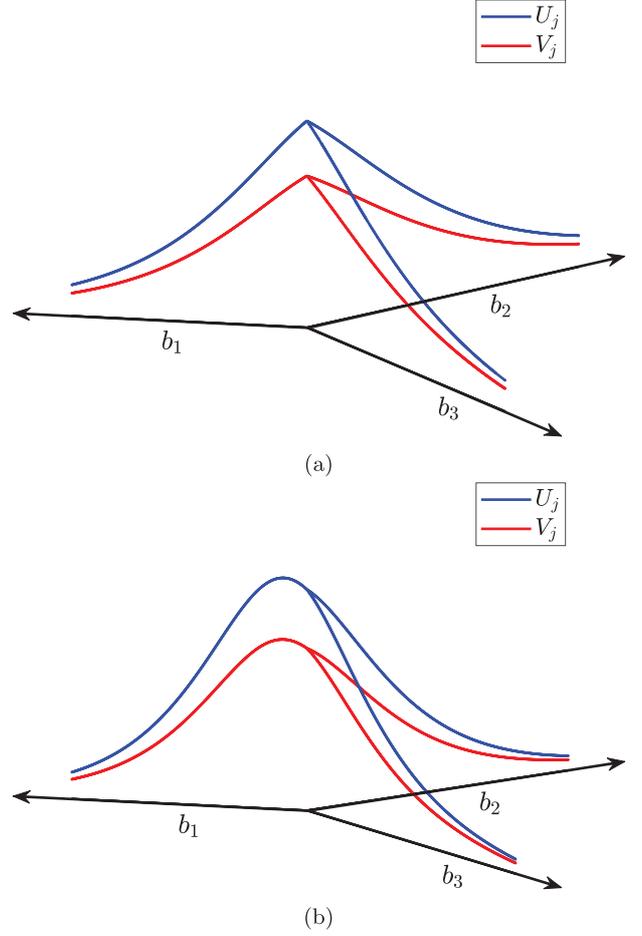


Fig. 4: Profile of standing Manakov solitons on a three-bond star graph plotted using the solutions (5) for $\lambda < 0$. The sign of $\{a_1, a_2, a_3\}$ is chosen as follows: $\{-, -, -\}$ for panel (a), and $\{+, -, -\}$ for panel (b). In both cases, the values of the parameters β_j and μ are chosen as $\beta_j = 1$, $\mu = 0.1$. The parameter a is calculated using eq. (13).

where real valued λ defines the type of boundary condition, *i.e.*, for $\lambda = 0$, eq. (8) represents Neumann boundary conditions, while for $\lambda = \infty$ it becomes a set of Dirichlet conditions. Nonzero λ values give the Kirchhoff conditions, and α_j is a positive constant which will be defined later.

To construct the solution of eq. (6) on a star graph, we use the following prescription. First we require that the known solution of the stationary Manakov system on a line (3) should fulfill the vertex boundary conditions given by eqs. (7) and (8). Then representing the solution of eq. (9) in the form

$$\Psi_j(x) = \left(\frac{\sqrt{\mu_1}}{\sqrt{\mu_2}} \right) \sqrt{\frac{2}{\beta_j}} \operatorname{sech}(\sqrt{\mu}(x - a_j)), \quad (9)$$

we find that fulfilling the vertex boundary conditions in eqs. (7) and (8) can be satisfied if and only if the following

constraints are fulfilled:

$$\begin{aligned} \frac{\alpha_j}{\alpha_1} &= \sqrt{\frac{\beta_j}{\beta_1}}, \quad |a_1| = |a_2| = \dots = |a_N|, \\ \frac{1}{\beta_1} \tanh(\sqrt{\mu}a_1) + \frac{1}{\beta_2} \tanh(\sqrt{\mu}a_2) + \dots \\ &+ \frac{1}{\beta_N} \tanh(\sqrt{\mu}a_N) = \frac{\lambda}{\sqrt{\mu}}. \end{aligned} \quad (10)$$

Introducing the notation $a_j = \varepsilon_j a$, where $\varepsilon_j = \text{sgn}(a_j)$ and taking into account the bell-like shape of the solution (see figs. 3 and 4), eq. (10) can be rewritten as

$$\tanh(\sqrt{\mu}a) \sum_{j=1}^N \frac{\varepsilon_j}{\beta_j} = \frac{\lambda}{\sqrt{\mu}}. \quad (11)$$

In eq. (11) the sign of the summation should be the same sign of λ , where $a, \mu, \beta_j > 0$. Thus one can write standing ‘‘bright-bright’’ soliton solution of eq. (9) as

$$\Psi_j^{(\ell)}(x) = \sqrt{\frac{2}{\beta_j}} \begin{cases} \Psi(a, x), & j = 1, \dots, \ell, \\ \Psi(-a, x), & j = \ell + 1, \dots, N, \end{cases} \quad (12)$$

$$a = \frac{1}{\sqrt{\mu}} \text{atanh} \left[\frac{\lambda}{\sqrt{\mu}} \left(\sum_{j=1}^{\ell} \frac{1}{\beta_j} - \sum_{j=\ell+1}^N \frac{1}{\beta_j} \right)^{-1} \right], \quad (13)$$

where ℓ is the number of bumps, while $N - \ell$ is the number of tails, and $\Psi(a, x)$ is the solution of the Manakov system on a line and given by eq. (5).

Figures 3 and 4 present plots of the solution (12) for the star graph with three bonds. In all cases the values of parameters μ and β_j are fixed as $\mu = 0.1$, $\beta_j = 1$, where $j = 1, 2, 3$. The value of a is calculated using eq. (13). Equation (11) provides a relation between the weights, a_j and nonlinearity coefficients, β_j . Since the latter depends on the properties of the material (*e.g.*, optical properties for Kerr materials), it fixes a relation between the weights and material properties. Such (constraint) relation can be used for tuning of standing soliton properties in branched structures (*e.g.*, optical fiber networks).

As presented in figs. 3 and 4, the bell-shaped profile of the solitons, along with nonzero value of the parameter a_j , lead to different forms of stationary states of the Manakov system on the star graph with three bonds. Particularly, in fig. 3 with $\lambda > 0$, panel (a) shows two bumps on the bonds $b_{1,2}$ and a single tail on the bond b_3 , whereas panel (b) displays three bumps on all the bonds. In contrast, in fig. 4 with $\lambda < 0$, panel (b) shows three tails, while panel (a) shows a single bump on the bond b_1 and two tails on the bonds $b_{2,3}$. Similarly, one can obtain ‘‘dark-dark’’ soliton solution of eq. (9). For this one needs to use dark-dark soliton solution on a line derived in ref. [23].

Ground-state solutions and stability analysis. – An interesting and physically relevant solution of the time-independent Manakov system is the ground state, which is

the solution of minimal energy. Here, we find the ground-state solution and provide a stability analysis using the same approach as in ref. [32]. Important characteristics for stability analysis and the ground-state solution are the norm and energy. The norm and the energy are given for the stationary Manakov system on metric star graph corresponding to the vertex boundary conditions in eqs. (7) and (8) (respectively) as

$$\begin{aligned} M &= \sum_{j=1}^N \int_0^{+\infty} (|U_j^2| + |V_j^2|) dx + \frac{\lambda \alpha_1^2}{2} (|U_1|^2 + |V_1|^2) \Big|_{x=0}, \\ E &= \sum_{j=1}^N \int_0^{+\infty} \left[\frac{1}{2} \left(\left| \frac{\partial U_j}{\partial x} \right|^2 + \left| \frac{\partial V_j}{\partial x} \right|^2 \right) - \frac{\beta_j}{4} (|U_j|^4 + |V_j|^4) \right. \\ &\quad \left. - \frac{\beta_j}{2} |U_j|^2 |V_j|^2 \right] dx + \frac{\lambda \alpha_1^2}{2} (|U_1|^2 + |V_1|^2) \Big|_{x=0}, \end{aligned} \quad (14)$$

where M and E are functions of $\Psi_j^{(\ell)}$ as $M = M[\Psi_j^{(\ell)}]$ and $E = E[\Psi_j^{(\ell)}]$

Substituting the solution in eq. (9) in the expression for the norm and the energy we get

$$\begin{aligned} M &= 2\sqrt{\mu} \sum_{j=1}^N \frac{1}{\beta_j} [\tanh(a_j \sqrt{\mu}) + 1] + \lambda \mu \text{sech}^2(a_1 \sqrt{\mu}), \\ E &= -\mu^{3/2} \sum_{j=1}^N \frac{1}{\beta_j} [\tanh(a_j \sqrt{\mu}) + 1] + \lambda \mu \text{sech}^2(a_1 \sqrt{\mu}). \end{aligned}$$

The stationary states of Manakov system on graphs previously determined are critical points ($S'_\mu[\Psi]$) of the so-called action functional S_μ [32]:

$$S_\mu = E[\Psi_j^{(0)}] + \frac{\mu}{2} M[\Psi_j^{(0)}]. \quad (15)$$

Stability can be analyzed using the Vakhitov-Kolokolov condition which (for stationary NLS equation on graph) can be written as

$$\frac{d^2}{d\mu^2} S_\mu = \frac{2\lambda \alpha_1^2}{\beta_1}. \quad (16)$$

One can see from eq. (16) that for $\lambda > 0$ we have $\frac{d^2}{d\mu^2} S_\mu > 0$, which means that the stationary solution in (12) is stable for all values of μ .

Extension to a tree graph. – Similarly to the above, one can extend the treatment to the case of other graph topologies, *e.g.*, for tree, loop, complete, or any other graphs. Here, we demonstrate this for the tree graph presented in fig. 5. The coordinate on each bond is assigned as follows: for bond b_1 we fix $x \in (\infty, 0]$, $x \in [0, L_{1j}]$ for bond b_{1j} , and $x \in [L_{1\Gamma}, L_{1\Gamma j}]$ for bond $b_{1\Gamma j}$, where $j = 1, 2$, $\Gamma = \{1, 2, 11, 12, 21, 22, \dots\}$, L_{1j} and $L_{1\Gamma j} - L_{1\Gamma}$ are the lengths of the bonds b_{1j} and $b_{1\Gamma j}$, respectively.

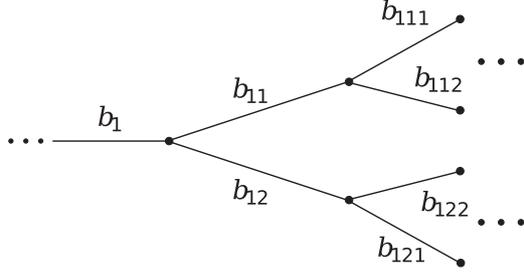


Fig. 5: Tree graph.

The stationary Manakov system is written on each bond of the tree graph as

$$\begin{aligned}
 \frac{d^2 U_1}{dx^2} + \beta_1(|U_1|^2 + |V_1|^2)U_1 &= \mu U_1, \\
 \frac{d^2 V_1}{dx^2} + \beta_1(|U_1|^2 + |V_1|^2)V_1 &= \mu V_1, \\
 \frac{d^2 U_{1\Gamma}}{dx^2} + \beta_{1\Gamma}(|U_{1\Gamma}|^2 + |V_{1\Gamma}|^2)U_{1\Gamma} &= \mu U_{1\Gamma}, \\
 \frac{d^2 V_{1\Gamma}}{dx^2} + \beta_{1\Gamma}(|U_{1\Gamma}|^2 + |V_{1\Gamma}|^2)V_{1\Gamma} &= \mu V_{1\Gamma}.
 \end{aligned} \tag{17}$$

The vertex boundary conditions can be written similarly to those for star graph, *i.e.*, in the form of continuity of the weight of solution and Kirchoff rules at each vertex:

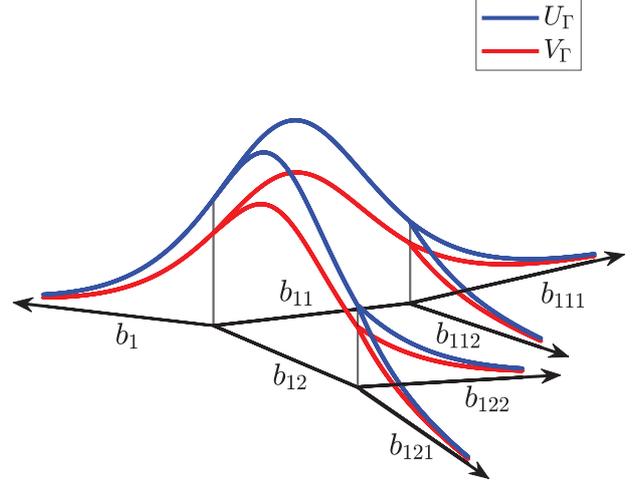
$$\begin{aligned}
 \alpha_1 \Psi_1|_{x=0} &= \alpha_{11} \Psi_{11}|_{x=0} = \alpha_{12} \Psi_{12}|_{x=0}, \\
 \frac{1}{\alpha_1} \frac{d\Psi_1}{dx} \Big|_{x=0} &+ \frac{1}{\alpha_{11}} \frac{d\Psi_{11}}{dx} \Big|_{x=0} + \frac{1}{\alpha_{12}} \frac{d\Psi_{12}}{dx} \Big|_{x=0} = \\
 \lambda \alpha_1 \Psi_1|_{x=0}, \\
 \alpha_{1\Gamma} \Psi_{1\Gamma}|_{x=L_{1\Gamma}} &= \alpha_{1\Gamma 1} \Psi_{1\Gamma 1}|_{x=L_{1\Gamma}} = \alpha_{1\Gamma 2} \Psi_{1\Gamma 2}|_{x=L_{1\Gamma}}, \\
 -\frac{1}{\alpha_{1\Gamma}} \frac{d\Psi_{1\Gamma}}{dx} \Big|_{x=L_{1\Gamma}} &+ \frac{1}{\alpha_{1\Gamma 1}} \frac{d\Psi_{1\Gamma 1}}{dx} \Big|_{x=L_{1\Gamma}} \\
 + \frac{1}{\alpha_{1\Gamma 2}} \frac{d\Psi_{1\Gamma 2}}{dx} \Big|_{x=L_{1\Gamma}} &= \lambda \alpha_{1\Gamma} \Psi_{1\Gamma}|_{x=L_{1\Gamma}},
 \end{aligned} \tag{18}$$

where $\Psi_1(x) = \begin{pmatrix} U_1(x) \\ V_1(x) \end{pmatrix}$ and $\Psi_{1\Gamma}(x) = \begin{pmatrix} U_{1\Gamma}(x) \\ V_{1\Gamma}(x) \end{pmatrix}$.

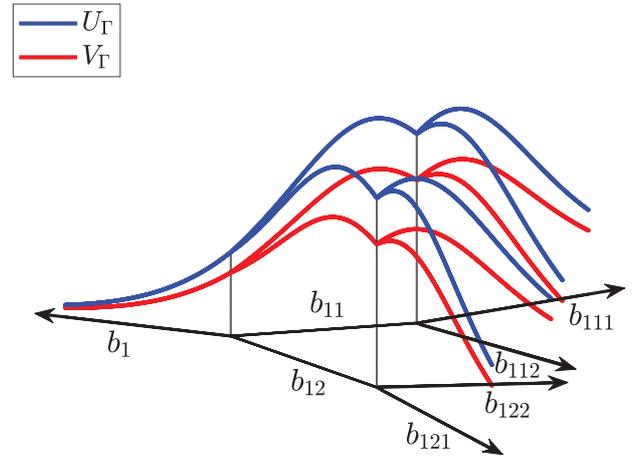
The solution of eq. (17) can be constructed from that for a line as

$$\begin{aligned}
 \Psi_1(x) &= \begin{pmatrix} \sqrt{\mu_1} \\ \sqrt{\mu_2} \end{pmatrix} \sqrt{\frac{2}{\beta_1}} \operatorname{sech}(\sqrt{\mu}(x - a_1)), \\
 \Psi_{1\Gamma}(x) &= \begin{pmatrix} \sqrt{\mu_1} \\ \sqrt{\mu_2} \end{pmatrix} \sqrt{\frac{2}{\beta_{1\Gamma}}} \operatorname{sech}(\sqrt{\mu}(x - a_{1\Gamma})), \\
 \Psi_{1\Gamma j}(x) &= \begin{pmatrix} \sqrt{\mu_1} \\ \sqrt{\mu_2} \end{pmatrix} \sqrt{\frac{2}{\beta_{1\Gamma j}}} \operatorname{sech}(\sqrt{\mu}(x - a_{1\Gamma j})).
 \end{aligned} \tag{19}$$

Fulfilling the solution of the vertex boundary conditions



(a)



(b)

Fig. 6: Profile of standing Manakov solitons on a tree graph plotted using the solutions (19) for $\Gamma = \{1, 2, 11, 12, 21, 22\}$ and for the signs of the parameters $\{a_1, a_{1j}, L_{1j} - a_{1j}, L_{1j} - a_{1j}\}$ ($i, j = 1, 2$): $\{-, +, +, +, +, -, -, -, -\}$ (a), and $\{-, +, +, +, +, +, +, +, +\}$ (b). For both cases $\beta_1 = \beta_{1\Gamma} = 1$ and $\mu = 0.1$.

leads to the following constraints:

$$\begin{aligned}
 \frac{\alpha_{1j}}{\alpha_1} &= \sqrt{\frac{\beta_{1j}}{\beta_1}}, \quad |a_1| = |a_{1j}|, \quad j = 1, 2, \\
 \frac{1}{\beta_1} \tanh(\sqrt{\mu}a_1) &+ \frac{1}{\beta_{11}} \tanh(\sqrt{\mu}a_{11}) \\
 + \frac{1}{\beta_{12}} \tanh(\sqrt{\mu}a_{12}) &= \frac{\lambda}{\sqrt{\mu}}, \\
 \frac{\alpha_{1\Gamma j}}{\alpha_{1\Gamma}} &= \sqrt{\frac{\beta_{1\Gamma j}}{\beta_{1\Gamma}}}, \quad |L_{1\Gamma} - a_{1\Gamma}| = |L_{1\Gamma} - a_{1\Gamma j}|, \quad j = 1, 2, \\
 \frac{1}{\beta_{1\Gamma}} \tanh[\sqrt{\mu}(L_{1\Gamma} - a_{1\Gamma})] &- \frac{1}{\beta_{1\Gamma 1}} \tanh[\sqrt{\mu}(L_{1\Gamma} - a_{1\Gamma 1})] \\
 - \frac{1}{\beta_{1\Gamma 2}} \tanh[\sqrt{\mu}(L_{1\Gamma} - a_{1\Gamma 2})] &= \frac{\lambda}{\sqrt{\mu}}.
 \end{aligned} \tag{20}$$

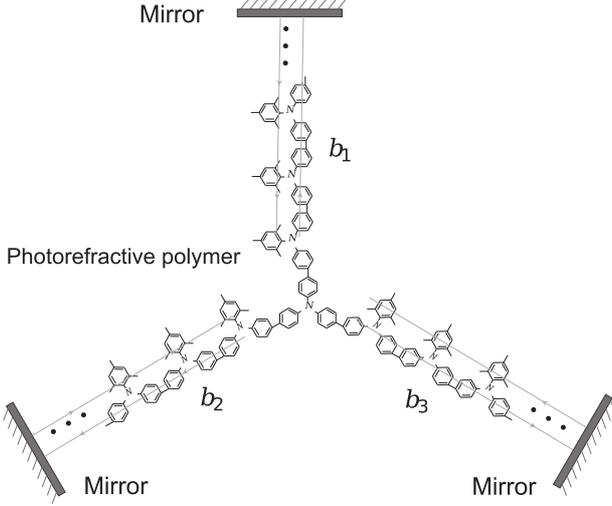


Fig. 7: A sketch for experimental realization of the standing Manakov solitons in a star-branched graph.

Introducing the notations $a_1 = \varepsilon_1 b_1$, $a_{1j} = \varepsilon_{1j} b_1$, $L_{1\Gamma} - a_{1\Gamma} = \bar{\varepsilon}_{1\Gamma} b_{1\Gamma}$, $L_{1\Gamma} - a_{1\Gamma j} = \bar{\varepsilon}_{1\Gamma j} b_{1\Gamma}$, where $\varepsilon_1 = \text{sgn}(a_1)$, $\varepsilon_{1j} = \text{sgn}(a_{1j})$, $\bar{\varepsilon}_{1\Gamma} = \text{sgn}(L_{1\Gamma} - a_{1\Gamma})$, $\bar{\varepsilon}_{1\Gamma j} = \text{sgn}(L_{1\Gamma} - a_{1\Gamma j})$ and $j = 1, 2$, eq. (20) can be rewritten as

$$\tanh(\sqrt{\mu} b_1) \left[\frac{\varepsilon_1}{\beta_1} + \frac{\varepsilon_{11}}{\beta_{11}} + \frac{\varepsilon_{12}}{\beta_{12}} \right] = \frac{\lambda}{\sqrt{\mu}},$$

$$\tanh(\sqrt{\mu} b_{1\Gamma}) \left[\frac{\bar{\varepsilon}_{1\Gamma}}{\beta_{1\Gamma}} - \frac{\bar{\varepsilon}_{1\Gamma 1}}{\beta_{1\Gamma 1}} - \frac{\bar{\varepsilon}_{1\Gamma 2}}{\beta_{1\Gamma 2}} \right] = \frac{\lambda}{\sqrt{\mu}}.$$

From the above equation, we can easily find b_1 and $b_{1\Gamma}$ as

$$b_1 = \frac{1}{\sqrt{\mu}} \text{atanh} \left[\frac{\lambda}{\sqrt{\mu}} \left(\frac{\varepsilon_1}{\beta_1} + \frac{\varepsilon_{11}}{\beta_{11}} + \frac{\varepsilon_{12}}{\beta_{12}} \right)^{-1} \right],$$

$$b_{1\Gamma} = \frac{1}{\sqrt{\mu}} \text{atanh} \left[\frac{\lambda}{\sqrt{\mu}} \left(\frac{\bar{\varepsilon}_{1\Gamma}}{\beta_{1\Gamma}} - \frac{\bar{\varepsilon}_{1\Gamma 1}}{\beta_{1\Gamma 1}} - \frac{\bar{\varepsilon}_{1\Gamma 2}}{\beta_{1\Gamma 2}} \right)^{-1} \right]. \quad (21)$$

In fig. 6, profiles of the standing Manakov solitons on the tree graph for $\Gamma = \{1, 2, 11, 12, 21, 22\}$ are plotted for the values of a_1 , $a_{1\Gamma}$ and λ for panel (a): $a_1 = -a_{1j} = -3.18$, $a_{1ji} = 11.18$, $\lambda = 0.5$; and for panel (b): $a_1 = -a_{1j} = -4.68$, $a_{1ji} = 3.32$, $\lambda = 20$, where $i, j = 1, 2$. The values of the following parameters are fixed for both panels as $\beta_1 = \beta_{1\Gamma} = 1$, $L_{11} = L_{12} = 8$ and $\mu = 0.1$. The values of b_1 and $b_{1\Gamma}$ are calculated using eq. (21).

One can conclude comparing the plots in the two panels that, for the bell shaped profile of the standing Manakov soliton, the bumps appear on the bonds b_{11} and b_{12} only, while the tails are formed on all other bonds in panel (a). Unlike the plots in panel (a), in the plots presented in panel (b), the tail appears on the bond b_1 only, while bumps are formed on all other bonds.

A sketch for experimental realization. – The above model can be experimentally realized using the branched photorefractive polymers or photorefractive crystals. A standing wave can be provided by putting

mirrors at the end of each branch of a polymer (crystal). A sketch for such experiment is presented in fig. 7. Manakov solitons in such polymers have been discussed earlier in ref. [46]. These polymers can be synthesized in the branched form. The sketch consists of branched photorefractive polymers in the form of Y-junction having the finite-length branches. By putting in contact mirrors (or any type of reflectors of the optical soliton) at the end of each branch, one can achieve formation of the standing Manakov solitons in the system. In such scheme, the optical soliton can come through one of the bonds, and after its entrance one should close the branch by putting the mirror. Fulfillment of the constraints given by eq. (10) can be controlled by tuning of the optical refractive index of the material and other physical characteristics.

Conclusions. – In this work, we studied time-independent Manakov system on metric graphs by focusing on static soliton solutions. An approach similar to that used in ref. [32] is used for finding the soliton and ground-state solutions, as well as for the stability analysis. Constraints approving soliton solutions in terms of the solution of the Manakov system on a line are derived. The ground-state solution of the problem is found and the stability analysis is provided. The study is extended to the case of a tree graph. The above model can be useful for designing and engineering of functional optical materials with tunable properties and for the device optimization in optoelectronics.

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REFERENCES

- [1] AGRAWAL G. P., *Applications of Nonlinear Fiber Optics* (Academic Press, San Diego) 2001.
- [2] KIVSHAR Y. S. and AGRAWAL G. P., *Optical Solitons: From Fibers to Photonic Crystals* (Academic Press, San Diego) 2003.
- [3] MANAKOV S. V., *Sov. Phys. JETP*, **38** (1974) 248.
- [4] RADHAKRISHNAN R. and LAKSHMANAN M., *J. Phys. A: Math. Gen.*, **28** (1995) 2683.
- [5] RADHAKRISHNAN R., LAKSHMANAN M. and HIETARINTA J., *Phys. Rev. E*, **56** (1997) 2213.
- [6] KANNA T. and LAKSHMANAN M., *Phys. Rev. Lett.*, **86** (2001) 5043.

- [7] KANG J. U., STEGEMAN G. I., AITCHISON J. S. and AKHMEDIEV N., *Phys. Rev. Lett.*, **76** (1996) 3699.
- [8] SHEPPARD A. P. and KIVSHAR Y. S., *Phys. Rev. E*, **55** (1997) 4773.
- [9] ZHONG W.-P., BELI M. and MALOMED B., *Phys. Rev. E*, **92** (2015) 053201.
- [10] STEIGLITZ K., *Phys. Rev. E*, **60** (2000) 016608.
- [11] STEIGLITZ K., *Phys. Rev. E*, **63** (2001) 046607.
- [12] ABLOWITZ M. J., PRINARI B. and TRUBATCH A. D., *Inverse Probl.*, **20** (2004) 1217.
- [13] OHTA Y., WANG D. S. and YANG J., *Stud. Appl. Math.*, **127** (2011) 345.
- [14] FENG B. F., *J. Phys. A: Math. Theor.*, **47** (2014) 355203.
- [15] MANČIĆ A., BARONIO F., HADŽIEVSKI LJ, WABNITZ S. and MALUCKOV A., *Phys. Rev. E*, **98** (2018) 012209.
- [16] MUSAMMIL N. M., SUBHA P. A. and NITHYANANDAN K., *Phys. Rev. E*, **100** (2019) 012213.
- [17] AKRAM G., SADAF M., ARSHED S. and FARRUKH M., *Opt. Quantum Electron.*, **56** (2024) 906.
- [18] LAKOBA T. I. and KAUP D. J., *Phys. Rev. E*, **56** (1997) 6147.
- [19] KAMCHATNOV A. M. and SOKOLOV V. V., *Phys. Rev.*, **91** (2015) 043621.
- [20] ZAHRAN E. H. M., BEKIR A. and IBRAHIM R. A., *Braz. J. Phys.*, **54** (2024) 171.
- [21] KATTI A., *Phys. Lett. A*, **458** (2023) 128590.
- [22] YAN-LI S., QI-CHANG J., XUAN-MANG J. and SHI-JIE X., *Commun. Theor. Phys.*, **57** (2012) 281.
- [23] FENG H. C., GUANG D. C., ABDURUSUL and QUN L. S., *Chin. Phys. Lett.*, **19** (2002) 63.
- [24] GANIE A. H., RAHAMAN MD. S., ALADSANI F. A. and ULLAH M. S., *Nonlinear Dyn.*, **113** (2025) 9807.
- [25] VIJAYAJAYANTHI M., KANNA T., MURALI K. and LAKSHMANAN M., *Phys. Rev. E*, **97** (2018) 060201(R).
- [26] VIJAYAJAYANTHI M., KANNA T., LAKSHMANAN M. and MURALI K., *Commun. Nonlinear Sci. Numer. Simul.*, **36** (2016) 391.
- [27] VIJAYAJAYANTHI M., KANNA T. and LAKSHMANAN M., *Phys. Rev. E*, **108** (2023) 054213.
- [28] YANG J., *Phys. Rev. E*, **59** (1999) 2393.
- [29] YANG J., *Phys. Rev. E*, **65** (2002) 036606.
- [30] GNUTZMANN S., KEATING J. P. and PIOTET F., *Ann. Phys.*, **325** (2010) 2595.
- [31] YUSUPOV J. R., SABIROV K. K., EHRHARDT M. and MATRASULOV D. U., *Phys. Lett. A*, **383** (2019) 2382.
- [32] ADAMI R., CACCIAPUOTI C., FINCO D. and NOJA D., *EPL*, **100** (2012) 10003.
- [33] SOBIROV Z., MATRASULOV D., SABIROV K., SAWADA S. and NAKAMURA K., *Phys. Rev. E*, **81** (2010) 066602.
- [34] SOBIROV Z., MATRASULOV D., SAWADA S. and NAKAMURA K., *Phys. Rev. E*, **84** (2011) 026609.
- [35] ADAMI R., CACCIAPUOTI C., FINCO D. and NOJA D., *Rev. Math. Phys.*, **23** (2011) 4.
- [36] NOJA D., *Philos. Trans. R. Soc. A*, **372** (2014) 20130002.
- [37] UECKER H., GRIESER D., SOBIROV Z., BABAJANOV D. and MATRASULOV D., *Phys. Rev. E*, **91** (2015) 023209.
- [38] SABIROV K. K., RAKHMANOV S., MATRASULOV D. and SUSANTO H., *Phys. Lett. A*, **382** (2018) 1092.
- [39] SABIROV K. K., BABAJANOV D. B., MATRASULOV D. U. and KEVREKIDIS P. G., *J. Phys. A: Math. Theor.*, **51** (2018) 435203.
- [40] SABIROV K. K., AKRAMOV M. E., OTAJONOV R. S. and MATRASULOV D. U., *Chaos, Solitons Fractals*, **133** (2020) 109636.
- [41] AKRAMOV M., SABIROV K., MATRASULOV D., SUSANTO H., USANOV S. and KARPOVA O., *Phys. Rev. E*, **105** (2022) 054205.
- [42] MATRASULOV J. and SABIROV K., *Physica A*, **608** (2022) 128279.
- [43] AKRAMOV M. E., YUSUPOV J. R., EHRHARDT M., SUSANTO H., MATRASULOV D. U., *Phys. Lett. A*, **459** (2023) 128611.
- [44] YUSUPOV J. R., MATYOKUBOV KH. SH., EHRHARDT M. and MATRASULOV D. U., *Phys. Lett. A*, **479** (2023) 128928.
- [45] DONG X., YANG Q., SPIESS CH., BUCKLEW V. G. and RENNINGER W. H., *Phys. Rev. Lett.*, **125** (2020) 033902.
- [46] TSUTSUMI N., *Polym. J.*, **48** (2016) 571.