Lecture Notes in Mechanical Engineering

Andrey A. Radionov
Vadim R. Gasiyarov *Editors*

Proceedings of the 7th International Conference on Industrial Engineering (ICIE 2021)

Volume I

Springer

Investigation of String Vibrations of a Transporting Device

M. U. Musirov[1(✉)] and N. Yu. Kholmanov[2]

[1] Academy of Sciences of the Republic of Uzbekistan, 33, Durmon Yuli Street, Tashkent, Uzbekistan100047

[2] National University of Uzbekistan Named After Mirzo Ulugbek, 4, University street, Tashkent, Uzbekistan100174

Abstract. String and belt conveyors are widely used in the light industry and agriculture to transport products and flat goods, in particular, leather products, to the processing area of the technological machines. For these conveyors to work effectively and efficiently for a long time, it is necessary to correctly select the geometrical and kinematic parameters at the process of their design. The technological processing of leather semi-finished products affects the quality of leather products. Therefore, it is necessary to analyze the conditions of the processed material after each technological operation. Experimental research in the leather industry is aimed at solving complex many-sided problems, revealing rational solutions to technological processes in leather raw materials processing. The motion of the conveyor strings, stretched on cylindrical shafts with the grooves, is determined by the Mathieu equation. The parameters of the Mathieu equation depend on the speed of the conveyor strings and geometrical parameters of the system. To limit the amplitude of the transverse vibrational motion of the conveyor strings, it is necessary to select the kinematic and geometrical parameters so that they fall into the stability zone. These parameters are determined depending on the geometrical and kinematic parameters of the device.

Keywords: Conveying device · Strings · Flat material · Variable mass · Lateral vibrations · Uniform feed

1 Introduction

Earlier, we developed a transporting device for feeding flat materials into the processing zone between rotating working shafts [1]. It was stated from the literature publications that the physical and mechanical properties of semi-finished leather products vary depending on their moisture content [2–7].

It is known that smooth and uniform feed of the processed material to the working area is one of the important tasks in various industries. Owing to it, substantial amounts of raw materials are saved when solving the issue of eliminating defective materials and losses in the process of feeding into the processing zone on transporting devices [8].

A. A. Radionov and V. R. Gasiyarov (eds.), *Proceedings of the 7th International Conference on Industrial Engineering (ICIE 2021)*, Lecture Notes in Mechanical Engineering, https://doi.org/10.1007/978-3-030-85233-7_15

Articles [9–12] are devoted to solutions of contact interaction in two-roll modules. Mathematical models of roll contact curves, friction stresses, and contact stress distribution patterns are obtained in these publications.

In [13], the deformation properties of a semi-finished leather product processed between the squeezing rollers covered with moisture-extracting materials were experimentally determined. The influence of the feeding speed and the pressure of the squeezing rollers on the deformation of a leather semi-finished product in its topographic sections (shoulder, belly, and butt) was determined.

2 Research Methods

Four analytical methods were used to determine the parameters, typical for the conveyor belts, and to analyze the test results when determining the elastic modulus; they are Euler–Bernoulli methods and finite element methods for determining the flexural modulus of unstretched belts, and the Timoshenko and Mindlin–Reissner methods [14].

It was determined that the conveyor belt slippage depends on the diameter of the driveshaft and vertical stress in the contact area, and that the belt resistance to its vibration is low [15].

It is known that the process of the conveyor belt splice is based on the Mooney–Rivlin law [16].

In problems of this type, the change in string velocity with time plays an important role in dynamic behavior. The transverse vibrations of the belt were calculated by the Kirchhoff method using the differential equations of motion of the system [17, 18].

Velocity control in belt conveyors results in energy efficiency and stable operation of basic units of conveyor equipment [19].

A virtual energy storage model was proposed to reduce the energy consumption of belt conveyors. This model studied the reduction in energy consumption over time by controlling the speed of the conveyor belt, feeding speed of the product, and the speed of other parts of the equipment [20].

Mechanical damage can occur after a certain time of conveyor operation. This process was considered and the methods to assess the damage probability during the operation were described using existing statistical models [21].

3 Research Results

Let us consider the problem of the transverse vibration of a conveyor belt with uneven gravity.

Non-uniformly distributed vertical forces of gravity of flat material being transported along the conveyor strings stretched on cylindrical shafts of radius R act on a transporting belt. Consider small vibrations of this system (Fig. 1).

We obtain the law of mass distribution of flat material in the conveyor as:

In the device under consideration, the conveyor strings move at a constant speed.

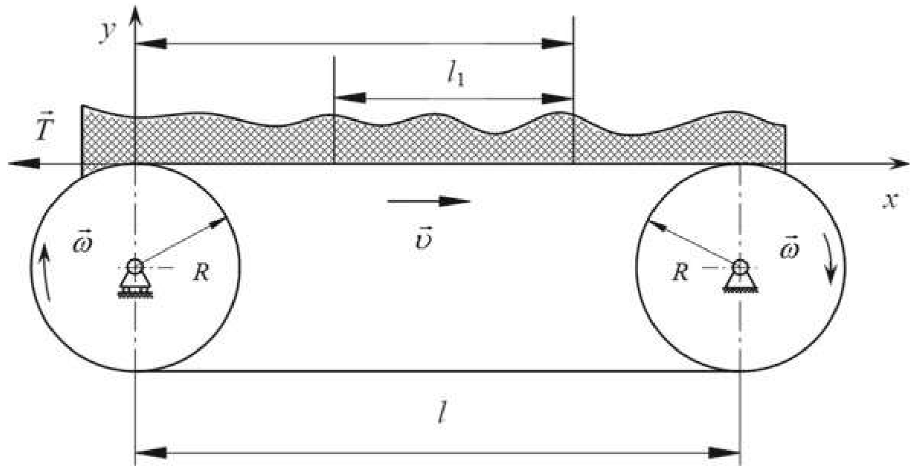

Fig. 1 Scheme of a device that moves flat materials to the processing area

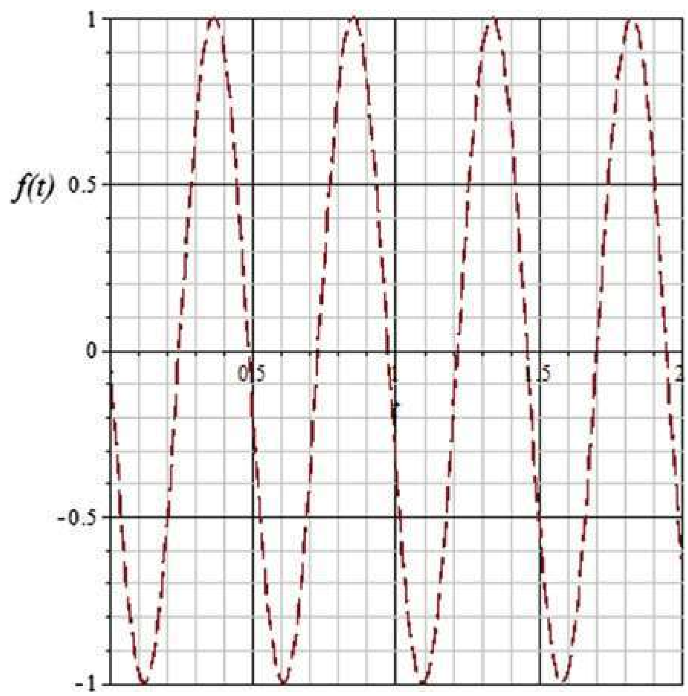

Fig. 2 Graphical solution of the Mathieu equation as a function of time

When solving problems of this type, it is convenient to consider them in the Euler coordinate system.

$$\frac{\mathrm{d}y}{dt} = \frac{\partial y}{\partial t} + \upsilon\frac{\partial y}{\partial x}, \frac{\mathrm{d}^2y}{\mathrm{d}t^2} = \frac{\partial^2 y}{\partial t^2} + 2\upsilon\frac{\partial^2 y}{\partial x\partial t} + \upsilon^2\frac{\partial^2 y}{\partial x^2}$$

Using the D'alembert principle, we write the following expression as [22, 23].

$$m\left(\frac{\partial^2 y}{\partial t^2} + 2\omega R\frac{\partial^2 y}{\partial x\partial t} + \omega^2R^2\frac{\partial^2 y}{\partial x^2}\right) = T\frac{\partial^2 y}{\partial x^2}.$$

In this case, m is the mass corresponding to the unit length

$$m = m_2 + m_0 + m_1 \sin\left(\frac{2\pi}{l_1}(x - \upsilon t)\right)$$

where m_0 is the average value of the mass of the transported product per unit length of the conveyor strings (kg/m); m_1—variable amplitude of the function of flat material mass (kg/m); m_2—mass per unit length of conveyor strings (kg/m); l_1—stepwise change in the variable mass of raw materials; T—tension of the working strings of the conveyor.

Since it is difficult to find a general analytical solution to the problem, we will use the Taylor series [24].

Assuming that m_1, i.e., the deviation of the non-uniformly distributed mass from m_0 is small, and then expanding the linear part $\frac{1}{m}$ around the point $m_1 = 0$ in the Taylor series, we obtain the equation of transverse motion in the following form

$$\frac{\partial^2 y}{\partial t^2} + 2\omega R \frac{\partial^2 y}{\partial x \partial t} + (\omega R)^2 \frac{\partial^2 y}{\partial x^2} = \frac{T}{m_0 + m_2}\left(1 - \frac{m_1}{m_0 + m_2} \sin\left(\frac{2\pi}{l_1}(x - \omega R t)\right)\right)\frac{\partial^2 y}{\partial x^2} \tag{1}$$

The initial and boundary conditions for the resulting partial product equation are as follows.

$$\begin{cases} y(0, t) = 0 \\ y(l, t) = 0 \end{cases}$$

Let us consider the problem of the small vibrations predominance by the Galerkin method.

$$y = f(t) \sin \frac{\pi x}{l} \tag{2}$$

It is easy to see that the approximating function from this point of view satisfies the boundary conditions. Substituting the relation (2) into the equation of motion (1), we obtain a simple second-order differential equation for the function *f(t)*.

$$\ddot{f} \sin\left(\frac{\pi x}{l}\right) + \frac{2\upsilon\pi}{l}\dot{f} \cos\left(\frac{\pi x}{l}\right) + \left(\frac{\pi}{l}\right)^2\left(\frac{T}{m_0 + m_2}\left(1 - \frac{m_1}{m_0 + m_2} \sin\left(\frac{2\pi}{l_1}(x - \omega R t)\right)\right) - \omega^2 R^2\right) \cdot \dot{f} \sin\left(\frac{\pi x}{l}\right) = 0 \tag{3}$$

To solve the differential Eq. (3), we construct the following system of differential equations.

$$\begin{cases} \frac{2\upsilon\pi}{l}\dot{f} = 0 \\ \ddot{f} + \left(\frac{\pi}{l}\right)^2\left(\frac{T}{m_0 + m_2}\left(1 - \frac{m_1}{m_0 + m_2} \sin\left(\frac{2\pi}{l_1}(x - \omega R t)\right)\right) - \omega^2 R^2\right) f = 0 \end{cases} \tag{4}$$

The first equation of the system of Eq. (4) always satisfies Eq. (1). We solve the second equation of the system

$$\ddot{f} + \left(\frac{\pi}{l}\right)^2 \left(\frac{T}{m_0 + m_2}\left(1 - \frac{m_1}{m_0 + m_2}\left(a_{11}\cos\frac{2\pi\omega Rt}{l_1} - a_{12}\sin\frac{2\pi\omega Rt}{l_1}\right)\right) - \omega^2 R^2\right) f = 0 \tag{5}$$

Here

$$a_{11} = \frac{l_1^2}{\pi^2\omega^2R^2}\int_0^l \sin\frac{2\pi x}{l_1}\sin^2\frac{\pi x}{l}\mathrm{d}x = -\frac{l_1^2}{\pi^2\omega^2R^2}\frac{l_1}{4\pi}\frac{l_1^2}{l^2 - l_1^2}\left(1 - \cos\frac{2\pi l}{l_1}\right)$$

$$a_{12} = \frac{l_1^2}{\pi^2\omega^2R^2}\int_0^l \cos\frac{2\pi x}{l_1}\sin^2\frac{\pi x}{l}\mathrm{d}x = -\frac{l_1^2}{\pi^2\omega^2R^2}\frac{l_1^2}{4\pi(l - l_1)}\sin\frac{2\pi l}{l_1}$$

The resulting equation is written as:

$$\ddot{f} + \left(a - b\sin\left(\frac{2\pi\omega Rt}{l_1} + \beta\right)\right)f = 0 \tag{6}$$

Here

$$a = \left(\frac{\pi}{l}\right)^2\left(\frac{T}{m_0 + m_2} - \omega^2R^2\right), b = \frac{Tm_1}{(m_0 + m_2)^2}\left(\frac{\pi}{l}\right)^2\sqrt{a_{11}^2 + a_{12}^2}, \beta = \mathrm{arctg}\left(\frac{a_{11}}{a_{12}}\right).$$

Let us introduce the following substitution:

$$\frac{2\pi\omega Rt}{l_1} = 2\tau - \frac{\pi}{2} - \beta$$

After this substitution, we compose the Mathieu equation.

$$\ddot{f} + (a + 2q\cos 2\tau)f = 0 \tag{7}$$

where

$$2q = b.$$

The general solution of the differential Eq. (7) is as follows:

$$f(t) = C_1\mathrm{MathieuC}(a, -q, \tau(t)) + C_2\mathrm{MathieuS}(a, -q, \tau(t))$$

Here.

MathieuC—is the Mathieu cosine, MathieuS—is the Mathieu sine;

C_1 and C_2 are the Mathieu's constant coefficients of differentiation, defined as follows:

$$C_1 = \frac{\mathrm{Mathieu}C(\mathrm{Mathieu}A(n,\ q),\ q,\ x)}{\mathrm{Mathieu}CE(n,\ q,\ x)}, C_2 = \frac{\mathrm{Mathieu}S(\mathrm{Mathieu}B(n,\ q),\ q,\ x)}{\mathrm{Mathieu}SE(n,\ q,\ x)}.$$

Here $n = 1{,}2{,}3\ldots.$

4 Discussion of Results

Problems of this type are reduced to the Mathieu equation by various methods. Solving the Mathieu equation obtained (6), it is convenient to determine and select the rational parameters of the working bodies of the transporting device of the technological machine.

We obtain a graphical solution of differential Eq. (6) for the given values $l_1 = 0.3$ m, $l = 1$ m, $T = 300$ N, $v = 0.2$ m/s, $m_0 = 0.15$ kg/m, $m_1 = 0.2$ kg/m, $m_2 = 0.3$ kg/m.

As seen from the obtained graphical solution of the Mathieu equation, the vibration amplitude of the conveyor strings does not change. If the amplitude does not change, the motion is stable. This ensures long-term and efficient operation of the working bodies of the transporting device.

5 Conclusions

The solution of the equation of motion (6) of the conveyor obtained for feeding flat materials to the processing zone is described by parametric vibrations depending on the values of the constants belonging to the system, considered in Fig. 1, where the amplitude of vibrations changes uniformly. An increase in the amplitude of vibrations with time leads to the occurrence of a parametric resonance state, and this process is considered undesirable for the mechanisms of the device. The movement of the conveyor strings stretched over the shafts, after several replacements, is determined by the Mathieu equation.

The parameters of the Mathieu equation depend on the speed of the conveyor strings and the geometrical parameters of the system. Since the amplitude of the oscillatory motion of the conveyor strings is limited, the kinematic and geometrical parameters must be selected so that they fall into the stability zone. These parameters depend on the geometry and speed of the device.

The values of the Mathieu equation solutions, representing the lateral vibrations of a mechanical system, were obtained using the Maple 18 programming package.

The computations obtained allow calculating the parameters of the transporting device of flat materials into the processing zone. They are useful for engineers when stating the operating modes of the conveyor in design of conveyors for various purposes. With technological requirements, we can set the speed and geometrical parameters of the transporting device. Its rational parameters can be chosen using the resulting equations.

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