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Non-Classical Theory of Rolling Using Asymmetric Technology

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Abstract. In industrial conditions during the rolling of thick plate, various asymmetries of the rolling process can occur, of which the most important are: uneven heating of the charge, mismatch between the neutral axis of the strip and the neutral axis of the deformation zone, unequal diameters of the upper and lower working rollers, inaccurately executed parts of the rolling cage, differences in the characteristics of the control systems for the drives of certain rollers. Nowadays, the optimization of energy-saving technologies in mechanical engineering to produce rolled sheets of different configurations using modern methods of mathematical simulation becomes a relevant direction for the informatization of machine-building production. At the same time, in the active zone, the process of cold rolling of metal sheets by asymmetric technology is accompanied by a nonhomogeneous stress-strain state. In this study, based on mathematical modeling in non-canonical deformation domains, an approach is proposed for determining optimal technological parameters when studying the stress-strain state of the cold rolling process using asymmetric and symmetric technologies in the active zone of elastoplastic strains.
Keywords: asymmetric technology, cold rolling, elastoplastic strains, stress function, metal strip.

INTRODUCTION

In an asymmetric rolling process, there are two zones of metal flow in the deformation zone. On the side of the metal delivery into the deformation zone, there is a forward slip zone, and on the exit, there is a backward slip zone. These zones are separated by a neutral plane, in which the metal speed is equal to the circular speed of the rollers. In the backward slip zone, the metal speed is less than the circular speed of the rollers but in the forward slip zone, this speed is higher [1]. In the case of a symmetric process, the length of these zones for both joint planes is the same. The introduction of asymmetry leads to an imbalance in the deformation zone; the lengths of certain zones and the location of the neutral zone change. In the deformation zone, on the side of each roller, zones of counter-directed contact stresses appear. The lengths of these zones depend on the magnitude of the introduced asymmetry [2-5].

PROBLEM STATEMENT

Let the body under consideration be a metal strip with thickness h and width b, which in the Ox_1x_3 plane when passing at a constant speed c between asymmetric driving and driven rollers at a distance $\delta = ct$ (t – time) experiences finite elastoplastic strain. In this case, the thickness decreases from $-h_n$ to $-h_k$ (Fig.1).

In the active rolling zone, an elementary rectangle for the cross-section of the strip under consideration, after the impact of the rollers under elastoplastic strains, transforms into a curved trapezoid, acquiring the contour of asymmetrical rollers. In the general case, for asymmetric technology, depending on the rolling conditions, the rotation speeds of the rollers can be different - $\omega_{\pm}$. Consequently, in the active rolling zone for the upper and lower surfaces of the metal strips, we have different speeds of displacements $c_{\pm} = R_{\pm}\omega_{\pm}$ [6-9].

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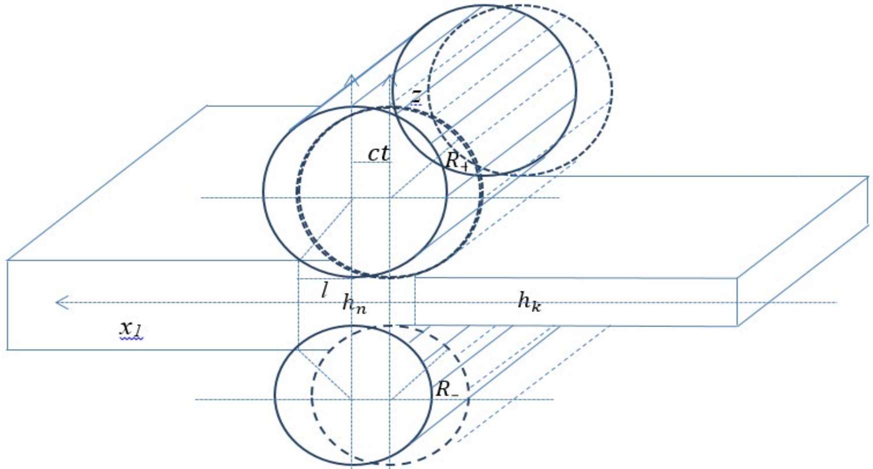

FIGURE 1. Spatial scheme of sheet rolling using asymmetric technology

During the rolling process, the metal strip moves at an arithmetic average speed. Let us calculate the arithmetic mean value of the speed of movement of the metal strip and the radii of the upper and lower rollers: $c = \frac{c_+ + c_-}{2}$, $R = \frac{R_+ + R_-}{2}$. A similar formula can be written for their differences: $\delta = \frac{c_+ - c_-}{2}$, $r = \frac{R_+ - R_-}{2}$. Therefore, $c_\pm = c \pm \delta$, $R_\pm = R \pm r$. For asymmetric technology, a straight metal strip, after passing through the active rolling zone, acquires an arcuate shape. In the case when the upper roller is driving, the lower one becomes the driven one, then $c_+ = c_- = c$ holds, and $\omega_- = \frac{R_+}{R_-}\omega_+$.

Let the total length of the metal strip under consideration be L. The equations of roller circles with radii $R_\pm$ in the active deformation zone relative to the moving strip at a certain time point are given by the following formulas:

$$(x_1 - ct \mp \delta t)^2 + (z - r \mp R \mp h_k/2)^2 = (R \pm r)^2.$$

To non-dimensionalize the participating variables, we relate the equations of circles to the arithmetic mean radius R, while leaving the notation unchanged:

$$(x_1 - ct \mp \delta t)^2 + (x_3 - r \mp 1 \mp h_k/2)^2 = (1 \pm r)^2.$$

The laws of change in the variable width of the upper and lower parts of the strip in the active zone of contact deformation have the following form:

$$h_\pm(x_1 - ct) = \pm\left[1 \pm r + \frac{h_k}{2} - \sqrt{(1 \pm r)^2 - (x_1 - ct \mp \delta t)^2}\right],$$

therefore, the thickness and asymmetry index of the metal strip in the active rolling zone are specified by a variable function of coordinate $-x_1$ and time $-t$

$$h(x_1 - ct) = h_+(x_1 - ct) - h_-(x_1 - ct) = \left[1 + r + \frac{h_k}{2} - \sqrt{1 + 2r + r^2 - (x_1 - ct - \delta t)^2}\right] + \left[1 - r + \frac{h_k}{2} - \sqrt{1 - 2r + r^2 - (x_1 - ct + \delta t)^2}\right],$$

$$\eta(x_1 - ct) = h_+(x_1 - ct) + h_-(x_1 - ct) = \left[1 + r + \frac{h_k}{2} - \sqrt{1 + 2r + r^2 - (x_1 - ct - \delta t)^2}\right] - \left[1 - r + \frac{h_k}{2} - \sqrt{1 - 2r + r^2 - (x_1 - ct + \delta t)^2}\right].$$

In particular, the following formula can be written:

$$h_\pm(x_1 - ct) = \frac{h(x_1 - ct) \pm \eta(x_1 - ct)}{2}$$

Let us assume that the upper and lower rollers have the same radii: $R_+ = R_- = R$, and rotation speeds $c_+ = c_- = c$, i.e. r=0, $\delta = 0$. There, a symmetrical rolling technology holds:

$$h(x_1 - ct) = 2\left[1 + \frac{h_k}{2} - \sqrt{1 - (x_1 - ct)^2}\right], \eta(x_1 - ct) = 0.$$

The resulting formulas involve irrational expressions, considering that all variables are related to R, therefore $y=x_1$-ct and $|y| \ll 1$ holds; then using the expansion of functions in a power series $(1 \pm y)^m \approx 1 \pm my + \frac{m(m-1)}{2}y^2$ for variable thickness and asymmetry index, after some calculations, we obtain:

$$\begin{cases} h(y) = h_k + (\delta t)^2 - \frac{1}{4}(\delta t)^4 + \left(1 - \frac{3}{2}\delta^2 t^2\right) y^2 + \frac{1}{4}y^4 \\ \qquad +r\delta t[(-2 + \delta^2 t^2)y + y^3]\,, \\ \eta(y) = \mathrm{r}\left[h_k + (\delta t)^2 - \frac{1}{4}(\delta t)^4 + \left(1 - \frac{3}{2}\delta^2 t^2\right) y^2 + \frac{1}{4}y^4\right] \\ \qquad +\delta t[(-2 + \delta^2 t^2)y + y^3]\,, \end{cases} \tag{1}$$

In this case, the lengths of the active zones of elastic-plastic strains on the upper and lower surfaces of the strip for asymmetric rolling have the following form:

$$l_{\pm} = \sqrt{(h_n - h_k)\left(1 \pm r - \frac{h_n}{4} + \frac{h_k}{4}\right)}, \tag{2}$$

moreover, for the active rolling zone $x_1 \epsilon (ct - \delta t, l + ct + \delta t)$,$l=max(l_-,)$.

For stress and strain deviators, we obtain [11]:

$$\tau_{ij} = \sigma_{ij} - \sigma\delta_{ij}, \tau_{kk} = 0,\ \sigma = \frac{1}{3}\sigma_{kk}\,,$$

$$e_{ij} = \varepsilon_{ij} - \varepsilon\delta_{ij}, e_{kk} = 0,\ \varepsilon = \frac{1}{3}\varepsilon_{kk}\,. \tag{3}$$

The defining relations between the deviators of the stress and elastoplastic strain tensors have the following form [10]:

$$\tau_{ij} = 2\mu(1 - \omega)e_{ij}$$

where $\omega = 1 - \frac{\sigma_i}{2\mu\varepsilon_i}$,

$$\sigma_i = \frac{1}{\sqrt{6}}\sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)}\,,$$

$$\varepsilon_i = \sqrt{\frac{2}{3}}\sqrt{(\varepsilon_{11} - \varepsilon_{22})^2 + (\varepsilon_{22} - \varepsilon_{33})^2 + (\varepsilon_{33} - \varepsilon_{11})^2 + \frac{3}{2}(\varepsilon_{12}^2 + \varepsilon_{23}^2 + \varepsilon_{31}^2)}$$

are the intensities of stresses and strains. E – elasticity modulus, ν – Poisson's ratio, $\mu = \frac{E}{2(1+\nu)}$ -shear modulus. Beyond the elastic limit - ε_y, the shear modulus becomes a decreasing function of the strain intensity. Then $\mu(\varepsilon_i)$ can be represented in the following form:

$$\mu(\varepsilon_i) = \begin{cases} \mu \cdot (a - b{\varepsilon_i}^2) & for\ \varepsilon_i > \varepsilon_y \\ \mu & for\ \varepsilon_i \le \varepsilon_y \end{cases}$$

where $a, b-$ are unknown constants to be defined, which, using the relationship between the intensities of stresses and strains

$$\sigma_i = 2\mu(\varepsilon_i)\,\varepsilon_i$$

are determined from the following condition:

$$\sigma_i = \sigma_T \text{ for } \quad \varepsilon_i = \varepsilon_T, \quad \sigma_i = \sigma_y \text{ for } \varepsilon_i = \varepsilon_y$$

where $\sigma_y, \varepsilon_y, \sigma_T, \varepsilon_T$ are the elasticity and yield limits of structural materials, determined from experiments. Considering the last relations and conditions, the relationship between the intensities of the stress and strain tensors in the active rolling zone takes the following form:

$$\sigma_i = \left[\frac{\sigma_y}{\varepsilon_y} - \frac{{\varepsilon_i}^2 - \varepsilon_y^2}{{\varepsilon_T}^2 - \varepsilon_y^2}\left(\frac{\sigma_y}{\varepsilon_y} - \frac{\sigma_T}{\varepsilon_T}\right)\right]\ \varepsilon_i,$$

therefore, we have a rational expression for the A.A. Ilyushin plasticity function [10-12]:

$$\omega = \omega_0 \frac{\varepsilon_i^2 - \varepsilon_y^2}{\varepsilon_T^2 - \varepsilon_y^2}, \quad \omega_0 = \left(1 - \frac{\sigma_T}{2\mu\varepsilon_T}\right)$$

Thus, this problem in the active deformation zone is considered in the non-canonical domain and within the framework of small elastoplastic strains. Considering the non-canonicity of the strain domain, we introduce a new variable ς:

$$z = \frac{1}{2}\eta(y) + h(y)\varsigma, \quad \varsigma \in \left(-\frac{1}{2}, \frac{1}{2}\right). z = x_3$$

Then the kinematic relations between the deformation tensor and the components of the displacement vector take the Lagrange-Green form:

$$\varepsilon_{11} = U_{1,1}, \varepsilon_{22} = U_{2,2}, \quad \varepsilon_{zz} = \frac{U_{z,\varsigma}}{h(y)}, \quad \varepsilon_{iz} = \frac{1}{2}\left(\frac{U_{i,\varsigma}}{h(y)} + U_{i,\mathrm{x}}\right), i = 1,2 \tag{5}$$

The sought-for solution is presented in the following form [2]:

$$\begin{cases} U_i = u_i + \psi_i \varsigma - \Phi_1(\varsigma) A_i - \frac{3}{5}\Phi_2(\varsigma) B_i \\ U_z = w + V\varsigma - \Phi_1(\varsigma)\theta \end{cases} \quad (6)$$

where $\Phi_1(\varsigma) = \frac{h^2}{12}\left[1 - 12\varsigma^2\right]$, $\Phi_2(\varsigma) = \frac{h^3}{4}\left[1 - \frac{20}{3}\varsigma^2\right]\varsigma$

Thus, using the equations of state of elastoplastic bodies (4), kinematic relations (5), and expressions for displacement components (6), it is possible to write analytical expressions for calculating the components of the stress tensor:[7,13,14]

$$\sigma_{ij} = \frac{1}{3}(3\lambda + 2\mu\omega)(\varepsilon_{kk} + \varepsilon_{zz})\delta_{ij} + 2\mu(1-\omega)\varepsilon_{ij},$$
$$\sigma_{zz} = \frac{1}{3}(3\lambda + 2\mu\omega)(\varepsilon_{kk} + \varepsilon_{zz}) + 2\mu(1-\omega)\varepsilon_{zz}$$
$$\sigma_{iz} = 2\mu(1-\omega)\varepsilon_{iz},$$

hence

$$\sigma_{ij} = \left(\lambda + \frac{2}{3}\mu\omega\right)\left[u_{k,k} + h\varsigma\psi_{k},_{k} - \frac{1}{2}\Phi_1(\varsigma)C_{k,k} - \frac{3}{5}\Phi_2(\varsigma)D_{k,k} + \left(V + 2\theta z\right)\right]\delta_{ij}$$
$$+ \mu(1-\omega)\left[u_{i,j} + u_{j,i} + h\varsigma\left(\psi_{i},_{j} + \psi_{j},_{i}\right) - \frac{1}{2}\Phi_1(\varsigma)(C_{i,j} + C_{j,i}) - \frac{3}{5}\Phi_2(\varsigma)(D_{i,j} + D_{j,i})\right] \quad (7)$$

$$\sigma_{zz} = \left(\lambda + \frac{2}{3}\mu\omega\right)\left[u_{k,k} + \varsigma\psi_{k},_{k} - \frac{1}{2}\Phi_1(\varsigma)C_{k,k} - \frac{3}{5}\Phi_2(\varsigma)D_{k,k} + \left(V + 2h\theta\varsigma\right)\right] + 2\mu(1-\omega)\left(V + 2h\theta\varsigma\right) \quad (8)$$

$$\sigma_{iz} = \mu(1-\omega)\left[\psi_i + 2h\varsigma C_i + 3h^2\varsigma^2 D_i + \left(w + hV\varsigma - \frac{1}{2}\Phi_1(\varsigma)\theta\right),_{i}\right]. \quad (9)$$

The resulting expressions involve functions C_i, D_i, u_i , ψ_i , w , θ and V of x_1, x_2, determined from the boundary conditions on the front surfaces during the rolling process of the strips under consideration.

To obtain a closed system of resolving equations for unknowns u_i ,ψ_i, w ,V, θ, we introduce into consideration the following integral quantities of the components of the stress tensor:

$$N_{ij} = \int_{-0.5h}^{0.5h} \sigma_{ij} dz$$ - normal forces,

$$Q_i = \int_{-0.5h}^{0.5h} \sigma_{zi} dz$$ - shear forces,

$$M_{ij} = \int_{-0.5h}^{0.5h} \sigma_{ij} z dz$$ - internal bending moments.

The stressed state of the body under consideration in the Cartesian coordinate system $Ox_1x_2\,\xi$, in the absence of body forces $X_i = 0$, is described by the following equilibrium equation [13,15-17]:

$$\begin{cases} \sigma_{ij},_{j} + \frac{\sigma_{iz},_{\xi}}{h} = 0 \\ \sigma_{zj},_{j} + \frac{\sigma_{zz},_{\xi}}{h} = 0 \end{cases} \quad (10)$$

To obtain equilibrium equations for rolling metal strips, we perform the integration procedure (10) over ξ in the interval [-0.5; 0.5]: [1, 2, 4]

$$\begin{cases} N_{ij,j} + \sigma_{iz}\Big|_{-0,5}^{0,5} = 0 \\ M_{ij,j} - Q_i + h\,\xi\sigma_{iz}\Big|_{-0,5}^{0,5} = 0, \\ Q_{i,i} + \sigma_{zz}\Big|_{-0,5}^{0,5} = 0 \end{cases} \quad (11)$$

In the free section, before and after the impact of the roller (sections I, IV), relative to the components of the elastic stress tensor ($\omega = 0$), we have the following boundary conditions:

$$\sigma_{zi} = 0, \ \sigma_{zz} = 0 \ \text{ for } \quad \varsigma = \pm\frac{1}{2} \quad (12)$$

Hence, considering (9), we obtain:

$$C_i = -\frac{1}{2}V,_i\ ,\ D_i = -\frac{5}{3h^2}\left(w,_i + \psi_i - \frac{h^2}{6}\theta,_i\right). \tag{13}$$

and substituting into (6), we have

$$\begin{cases} U_i = u_i + \left[h\varsigma + \Phi_2(\varsigma)\right]\psi_i - \frac{1}{2}\Phi_1(\varsigma)V,_i - \Phi_2(\varsigma)\left(w + \frac{h^2}{6}\theta\right),_i \\ U_z = w + h\varsigma V - \Phi_1(\varsigma)\theta, \end{cases} \tag{14}$$

Considering (14) for normal stress σ_{zz}, we can obtain the following expressions:

$$\sigma_{zz} = \lambda\left[u_{k,k} + \left(\Phi_2(\varsigma) + h\varsigma\right)\psi_i,_i + \Phi_2(\varsigma)\Delta\left(w - \frac{h^2}{30}\theta\right) + \frac{1}{2}\Phi_1(\varsigma)\Delta V + \frac{1-\nu}{\nu}\left(V + 2h\theta\varsigma\right)\right] \tag{15}$$

Boundary condition (12) for σ_{zz} are met by performing the following partial differentiation:

$$\begin{cases} D\Delta V - 2G\left(u_{k,k} + \frac{1-\nu}{\nu}V\right) = 0, \\ D\Delta\theta - G\left(5\psi_k,_k - \Delta w\right) - 12\frac{1-\nu}{\nu}\theta = 0. \end{cases} \tag{16}$$

where $D = \frac{Eh^3}{12(1+\nu)}$, $G = \frac{Eh}{2(1+\nu)}$, Δ is the Laplace operator.

After satisfying boundary condition (12) with (9) and (13), for shear stresses, we have the following expression:

$$\sigma_{iz} = \frac{5}{4}\mu\left[\psi_i + \left(w + \frac{h^2}{6}\theta\right),_i\right]f(\varsigma), \quad f(\varsigma) = 1 - 4\varsigma^2 \tag{15}$$

The components of the displacement vector (14) and the corresponding components of the symmetric stress tensor depend on the unknown integral quantities u_i, ψ_i, w,V, θ, which are functions of coordinates x_1, x_2, and t.

Outside the active rolling zone, the integral expressions for internal forces and moments, introducing stress tensor components (7) and (15), are:

$$N_{ij} = \breve{G}\left\{\frac{2\nu}{1-2\nu}\left(u_{k,k} + V\right)\delta_{ij} + u_{i,j} + u_{j,i}\right\} \tag{16}$$

$$M_{ij} = \breve{D}\left[\frac{\nu}{1-2\nu}\left(\psi_k,_k + 2\theta\right)\delta_{ij} + \frac{1}{2}\left(\psi_i,_j + \psi_j,_i\right)\right], \tag{17}$$

$$Q_i = \breve{G}k^2\left[\psi_i + \left(w + \frac{h^2}{6}\theta\right),_i\right] \qquad k^2 = \frac{5}{6}. \tag{18}$$

It should be noted here that the so-called shear coefficient k^2 is determined as a result of the fulfillment of the boundary conditions for shear stresses. At the same time, in refined theories of the Timoshenko type, special experiments were conducted to determine this coefficient [18-21]. Experimental results are quite close to 5/6.

Equilibrium equations (11) considering (16)-(18), take the following form:

$$\begin{cases} \Delta u_i + \frac{1}{1-2\nu}u_{k,ki} + \frac{2\nu}{1-2\nu}V,_i = 0 \\ \Delta\psi_i + \frac{1}{1-2\nu}\psi_k,_{ki} + \frac{1+14\nu}{1-2\nu}\theta,_i - \frac{10}{h^2}\left(\psi_i + w,_i\right) = 0 \\ \psi_k,_k + \Delta w - \frac{h^2}{30}\Delta\theta = 0 \end{cases} \tag{19}$$

As an example to present the features of the proposed approach to mathematical modeling of rolling metal strips, we consider a symmetrical technology $c_+ = c_-, \delta = 0,\ R_+ = R_-, r = 0$. Then the intermediate (II-section) and active (III-section) zones merge $\left(,\ l_- = l_+ = l,\ h_\pm = \pm\frac{h}{2}\right)$. Due to the impact of the rollers, internal stresses arise on the surface of the metal strips, which satisfy the following boundary conditions:

$$\begin{cases} U_z = w_{\pm} \\ \sigma_{11}n_1 + \sigma_{1z}n_z = \sigma n_1 + \tau n_z, \\ \sigma_{z1}n_1 + \sigma_{zz}n_z = \tau n_1 + \sigma n_z, \end{cases} \quad \text{for} \quad \varsigma = \pm\frac{1}{2} \tag{22}$$

where $w_{\pm} = h_{\pm}(y+ct) - h_{\pm}(y) = \pm\frac{1}{2}\left[\sqrt{1-y^2} - \sqrt{1-(y+ct)^2}\right]$,

Considering the expression for normal displacement in (6) from the boundary condition (22), we obtain:

$$w + \frac{h^2}{6}\theta = 0, \quad hV = \left[\sqrt{1-y^2} - \sqrt{1-(y+ct)^2}\right] \tag{23}$$

Then for normal displacement, we obtain:

$$U_z = w\left(1-4\varsigma^2\right) + \left[\sqrt{1-y^2} - \sqrt{1-(y+ct)^2}\right]\varsigma \tag{24}$$

RESULTS

In the problem under consideration, the external shear stress τ arises due to the roller interaction with the metal strip as the Coulomb friction force from the normal pressure σ, i.e. $\tau = k\sigma$ considering the plane-parallel translational motion of rollers, $n_2 = 0$. The normal impact of the rollers in the active zone σ is the function of the angle: $\sigma = \pm \mathrm{P} sin\alpha_{\pm}$, where P − is the pressure force from the rollers on the surface of the metal strip in the active rolling zone. Provided that the length of the active rolling zone is small relative to the total length and the difference between the initial and final thickness of the metal strip is small, we obtain $\frac{n_1}{n_z} = tg\alpha_{\pm} = h'_{\pm}(y) << 1$, therefore, $\sin\alpha_{\pm} \approx \alpha_{\pm} \approx tg\alpha_{\pm} \approx h'_{\pm}(y)$, hence, we obtain $\sigma = \pm Ph'_{\pm}(y)$. Resolving the last two equations (22) with respect to σ_{1z}, σ_{zz} and ignoring quantities of the high-order smallness, we obtain a simplified formula for the boundary condition in the active zone of rolling a metal strip:

$$\begin{cases} \sigma_{1z} = kPh'_{\pm} \\ \sigma_{zz} = Ph'_{\pm} \end{cases} \quad \text{for} \quad \varsigma = \pm\frac{1}{2} \tag{25}$$

Substituting expressions for shear and normal stresses from (8) and (9) into (25), we obtain:

$$C_1 = \kappa Ph' - \frac{1}{2}V,_1, \; D_1 = -\frac{5}{3h^2}\left(w,_1 + \psi_1 - \frac{h^2}{6}\theta,_1\right). \tag{26}$$

$$\frac{h^2}{12}C_{2,2} = -u_{k,k} + \frac{h^2}{12}\left(\kappa Ph' - \frac{1}{2}V,_1\right) - \left[1 + \frac{(1-\omega)(1-2\nu)}{3\nu + (1-2\nu)\omega}\right]V \tag{27}$$

$$\frac{h^3}{10}D_{2,2} = -\frac{\mathrm{P}h'}{\left(\lambda + \frac{2}{3}\mu\omega\right)} + h\psi_k,_k + \frac{1}{2}\left(w,_1 + \psi_1 - \frac{h^2}{6}\theta,_1\right),_1 h + 2h\left[1 + \frac{(1-\omega)(1-2\nu)}{3\nu + (1-2\nu)\omega}\right]\theta \tag{28}$$

Then the equilibrium equation (11) with (16)-(18) and (25)-(28), takes the following form:

$$\begin{cases} N_{1j,j} + \kappa Ph' = 0, \quad N_{2j,j} + 2\mu(1-\omega)hC_2 = 0 \\ M_{1j,j} - Q_1 = 0, \quad M_{2j,j} - Q_2 + h\mu(1-\omega)\left[\psi_2 + \frac{3}{4}h^2\varsigma^2 D_2\right] = 0 \\ Q_{i,i} + Ph' = 0, \end{cases} \tag{28}$$

The boundary conditions corresponding to different zones ensuring continuity (Fig. 2) of the process of symmetrical rolling along the Ox_1 axis can be formulated as follows:

$$\begin{cases} \sigma_{z1} = 0, \; \sigma_{21} = 0, \sigma_{21} = 0 & \text{for} \quad \mathrm{x}_1 = -\mathrm{ct}, \\ U_1|_{IV} = U_1|_{III}, U_2|_{IV} = U_2|_{III}, U_z|_{IV} = U_z|_{III} & \text{for} \quad \mathrm{x}_1 = 0, \\ \sigma_{z1}|_{IV} = \sigma_{z1}|_{III}, \sigma_{21}|_{IV} = \sigma_{21}|_{III}, \sigma_{21}|_{IV} = \sigma_{21}|_{II}\mathrm{I} & \end{cases} \tag{29}$$

$$\begin{cases} U_1|_{III} = U_1|_{I}, U_2|_{III} = U_2|_{I}, U_z|_{III} = U_z|_{I} & \text{for} \quad \mathrm{x}_1 = \mathrm{l} - \mathrm{ct}, \\ \sigma_{z1}|_{III} = \sigma_{z1}|_{I}, \sigma_{21}|_{III} = \sigma_{21}|_{I}, \sigma_{21}|_{III} = \sigma_{21}|_{I} & \\ \sigma_{z1} = 0, \; \sigma_{21} = 0, \sigma_{21} = 0 & \text{for} \quad \mathrm{x}_1 = \mathrm{L} \end{cases} \tag{30}$$

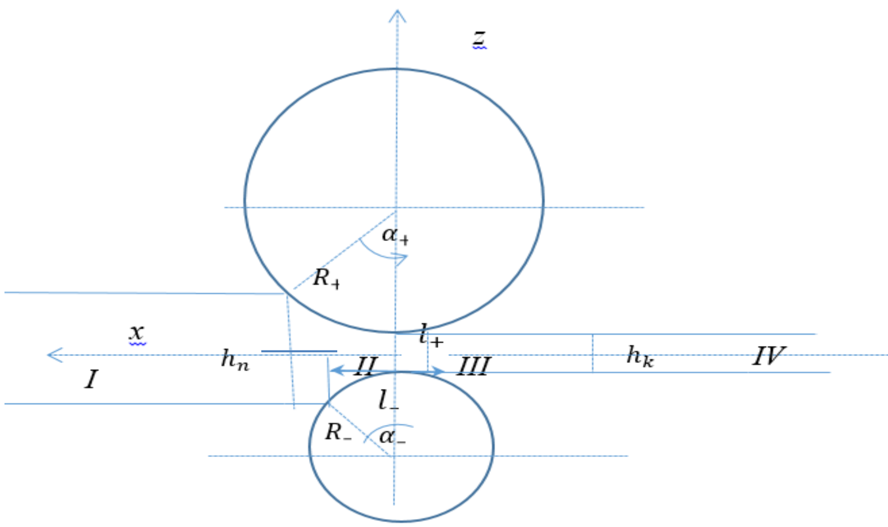

FIGURE 2 Transition zones for asymmetric technology:
I - free zone before rolling $y \geq l_1$; II - intermediate pre-rolling zone, $y\epsilon(l_2, l_1)$;
III — active rolling zone, $y\epsilon(0, l_2)$; IV – free zone after rolling y≤0.

Along the $0x_2$ axis, the boundary conditions corresponding to the free edge are met.

Thus, we have a mathematical model of elastic-plastic strain during the rolling of a metal strip using symmetric and asymmetric technology.

CONCLUSION

1. A non-classical theory of rolling metal strips using symmetric and asymmetric technologies is proposed.
2. A rational formula for elastic-plastic strain between the intensities of stress and strain deviators was obtained.

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